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Glauber-Gribov approach for DIS on nuclei in N=4 SYM

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Abstract: In this paper the Glauber-Gribov approach for deep-inelastic scattering (DIS) with nuclei is developed in N=4 SYM. It is shown that the amplitude displays the same general properties, such as geometrical scaling, as is the case in the high density QCD approach. We found that the quantum effects leading to the graviton reggeization, give rise to an imaginary part of the nucleon amplitude, which makes the DIS in N=4 SYM almost identical to the one expected in high density QCD. We concluded that the impact parameter dependence of the nucleon amplitude is very essential for N=4 SYM, and the entire kinematic region can be divided into three regions which are discussed in the paper. We revisited the dipole description for DIS and proposed a new renormalized Lagrangian for the shock wave formalism which reproduces the Glauber-Gribov approach in a certain kinematic region. However the saturation momentum turns out to be independent of energy, as it has been discussed by Albacete, Kovchegov and Taliotis. We discuss the physical meaning of such a saturation momentum $Q_s(A)$ and argue that one can consider only $Q > Q_s(A)$ within the shock wave approximation.

Keywords: QCD Phenomenology, Phenomenology of Field Theories in Higher Dimensions.

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1. Introduction

The goal of this paper is very modest and pragmatic: to write a Glauber-type formula for deep inelastic scattering (DIS) with a nucleus in N=4 SYM. N=4 SYM at weak couplings is similar to our microscopic theory of QCD, with gauge colour group SU(N_c). The high energy amplitude in this theory is given by the exchange of the BFKL Pomeron, like in QCD [1]. On the other hand, the AdS/CFT correspondence [2] allows us to calculate this amplitude in the strong coupling limit, where it reveals a Regge behavior (see ref. [3 – 5] and references therein). Therefore, in principle, considering the high energy scattering amplitude in N=4 SYM, we can guess what physics phenomena could be important in QCD, in the limit of strong coupling.

The simplest and most informative process that allows to study physics in the region between short distances and long distances, is DIS in the wide range of photon virtualities Q. Since the typical distances are $r \propto 1/Q$, we can approach the long distance physics at small values of Q. In QCD, we see three different regions for DIS:

1. $Q^2 \gg Q_s^2(x)$ where $Q_s^2(x)$ is the new scale: saturation momentum (see refs. [7 – 9] and a short but beautiful review in ref. [10]). At such large Q^2, we can use a linear evolution equation, namely the DGLAP equation [11], and the BFKL equation [12], and all advantages of the operator Product Expansion [13].

2. $\Lambda^2_{\rm QCD} \ll Q^2 \ll Q_s^2(x)$. In this region the density of partons (gluons) is so large that we cannot use here the methods of perturbative QCD. However, the QCD couplings are still small here, since the typical distances in this kinematic region are $r \propto 1/Q_s(x)$, and $Q_s(x) \gg \Lambda_{\rm QCD}$. This fact allows us to suggest a theoretical approach in this region, based on non-linear equations [14 – 16].

3. $Q^2 \leq \Lambda^2_{\rm QCD}$. No rigorous theoretical approach has been developed in this region in QCD. In high energy phenomenology, we describe this region with the soft Pomeron. However, quite a different phenomenological approach has been tried in this region, namely, that the saturation scale determines the physics inside this domain, and instead of the soft Pomeron, we can use the scattering amplitude in the saturation region (see refs. [17, 18]). Our intention is to use the input from our N=4 SYM experience, to penetrate this domain.

It turns out that N=4 SYM leads to normal QCD like physics in the first region, with OPE and linear equations (see refs. [19]). It has been shown in ref. [10] that the DIS densities reach saturation at some value of momentum ($Q_s(x)$). However, the physical picture inside the saturation domain turns out to be completely different [10], in the sense that there are no partons in this region and the main contribution stems from diffractive processes when the target (proton) either remains intact, or is slightly excited. Such a picture not only contradicts the common wisdom, but also contradicts available experimental data.

In this paper we would like to develop a systematic approach to DIS with a nucleus, based on the eikonal formula. In QCD the most reliable approach has been developed for this particular case, since a new parameter appears $\alpha_s A^{1/3} \approx 1$, which allows to prove the non-linear equation [15].

2. Eikonal approximation for scattering with nuclei.

2.1 General approach

It is well known that the eikonal approach is based on two main ideas [20, 21]. The first one is the fact that the value of typical impact parameter for the interaction with a proton is much smaller than the typical impact parameter for the nucleon distribution in a nucleus. Using this idea, we can easily express the amplitude for interaction with a nucleus via the interaction amplitude with a nucleon. Indeed, let us consider a simple example when the amplitude of interaction with the nucleon is small. Consider for example deep inelastic scattering (DIS) with a nucleon. The DIS amplitude for the virtual photon (γ^*) interaction with the nuclear target (A), can be written as follows

$$A\left(\gamma^* A; s, b\right) \;=\; \int d^2b'\, A\left(\gamma^* N; s, b'\right)\, S\left(\vec{b} - \vec{b}'\right) \;\longrightarrow\; \int d^2b'\, A\left(\gamma^* N; s, b'\right)\, S\left(b\right) \quad (2.1)$$

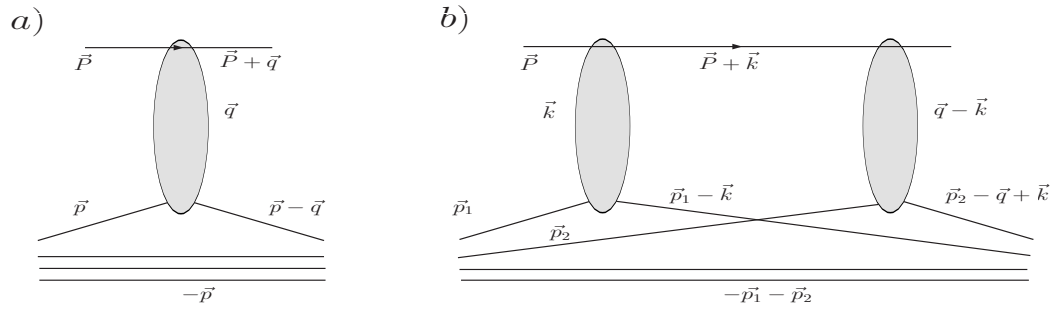

Figure 1: The single (figure 1-a) and double (figure 1-b) rescattering with heavy nucleus.

where $S(b)$ is the distribution of nucleons in the nucleus, normalized as $\int d^2b\, S(b) = A$, where A is the number of nucleons in a nucleus. In eq. (2.1) we use the fact that $|\vec{b}-\vec{b}'| \approx R_A \gg R_N \approx b'$. R_A is the nucleus radius while R_N is the nucleon size. $\int d^2b'\, A(\gamma^* N; s, b')$ is equal to the forward scattering amplitude $A_N(s, t=0)$. In the original Glauber-Gribov approach it was assumed that $A_N(s, t=0)$ at high energy is mostly imaginary, and $ImA_N(s, t=0) = \sigma^N_{\rm tot}$ [1]

The second important observation is the fact that at high energies the longitudinal and transverse degrees of freedom are factorized in such a way, that in first approximation the interactions with many nucleons in a nucleus will affect the transverse degrees of freedom and the impact parameter distribution, but we can neglect the feedback of these interactions on the momentum and the trajectory of the fast projectile. In other words, we can use the eikonal approximation for high energy scattering.

To illustrate this point, let us consider the interaction of the fast particle with the nucleus at rest, as it is shown in figure 1.

First we demonstrate that the momentum transferred q in figure 1, is transverse at high energy. For the nuclear target, it is preferable to discuss a process in the rest frame of a nucleus. Describing the nucleus in the non-relativistic approach, we consider that the kinetic energy of a nucleon is much smaller than its momentum, namely, $p^2/2m \ll |\vec{p}| \sim 1/R_A$ where $R_A \gg R_N$. Since after rescattering, the nucleon with momentum $\vec{p}-\vec{q}$ is still in the same nucleus, $q_0 = p_0 - (p-q)_0 = p^2/2m - |\vec{p}-\vec{q}|^2/2m \ll |\vec{q}|$. In our frame, $s = 2E\,M_A$ where E is the energy of the projectile, and M_A is the mass of the nucleus. At high energy, the momentum of the projectile is $P = (E, 0, 0, E)$. Using the fact that $P^2 = m_p^2$ and $(P+q)^2 = m_p^2$ where m_p is the mass of projectile, we obtain that

$$2P\cdot q = -q^2\,; \qquad q_0 - q_z = -q^2/2E \tag{2.2}$$

where z is the beam direction. Calculating q^2 we have

$$q^2 = (q_0+q_z)(q_0-q_z) - q_\perp^2 = -q^2/2E(2q_0 + q^2/2E) \xrightarrow{E\gg m} -q_\perp^2 \tag{2.3}$$

[1] It should be noticed that such normalization of the amplitude is a bit unusual for high energy physics since the amplitude, calculated from the Feynman diagrams, has a different normalization, namely, $ImA = s\,\sigma_{\rm tot}$. We call the first one as non-relativistic while the amplitude of Feynman diagrams will be called relativistic.

The expression for the diagram of figure 1-a has the following form

$$A_A\left(s,q^2\right) \;=\; \int \frac{d^4p_1}{(2\pi)^4 i}\,\frac{1}{m^2-p^2-i\epsilon}\,A_N\left(s,q_\perp^2;p_1^2,(p-q)^2\right);\,\frac{1}{m^2-(p-q)^2-i\epsilon}$$
$$\times\int\prod_{i=1}^{A-1}\frac{d^4p_i}{(2\pi)^4 i}\,\Gamma\left(p_1;\{p_i\}\right)\,\frac{1}{m^2-p_i^2-i\epsilon}\,\Gamma\left(p_1-q;\{p_i\}\right) \tag{2.4}$$

where $\Gamma\left(p_1;\{p_i\}\right)$ is the vertex for the transition of the nucleus into A free nucleons. Introducing a new variable for the energies of the nucleons, namely, $p_{0,i} \equiv M_A/A - \tilde{p}_{0,i}$ and noticing that since $\tilde{p}_{0,i}$ has the interpretation of being the kinetic energy, we anticipate very small values of $\tilde{p}_{0,i} \ll |\vec{p}_i|$, and therefore we can neglect $\tilde{p}_{0,i}^2$. Using this approach, each propagator has the form

$$\begin{aligned} m^2-p_i^2-i\epsilon \;&=\; \left(-\,\frac{M_A^2}{A^2}+m^2\right)\;+\;2\tilde{p}_{0,i}\,\frac{M_A}{A}\;+\;|\vec{p}_i|^2\;-\;i\epsilon \\ &=\; m\,\varepsilon+2\tilde{p}_{0,i}\,m\;+\;|\vec{p}_i|^2\;-i\epsilon\;\;\text{for } i<A \\ \text{but } m^2-p^2-i\epsilon \;&=\; \left(-\,\frac{M_A^2}{A^2}+m^2\right)-2\sum_{i=1}^{A-1}\tilde{p}_{0,i}\,\frac{M_A}{A}\;+\;|\vec{p}|^2-i\epsilon \\ &=\; m\,\varepsilon\;-2\;\sum_{i=1}^{A-1}\tilde{p}_{0,i}\,m\;+\;|\vec{p}|^2\;-\;i\epsilon \end{aligned} \tag{2.5}$$

where $\varepsilon = (M_A - Am)/A$ is the bounding energy per one nucleon in a nucleus, which is much smaller than the mass of the lightest hadron. One can see that all propagators for $i < A$, have poles in $\tilde{p}_{0,i}$ in the upper semi-plane, while the A-th propagator has a pole in the lower semi-plane. Closing the contour of integration over $\tilde{p}_{0,i}$, on the poles in the lower semi-plane, we obtain the following anticipated result, namely

$$A_A\left(s,q^2\right) \;=\; \int\prod_{i=1}^{A}\frac{d^3p_i}{(2\pi)^3}\,\Gamma\left(p_1;\{p_i\}\right)\,\frac{1}{A\,\varepsilon-\sum_{i=1}^{A}\frac{|\vec{p}_i|^2}{2m}-i\epsilon}\,A_N\left(s,q_\perp^2;p_1^2,(p-q)^2\right)$$
$$\times\,\frac{1}{A\,\varepsilon-\frac{(\vec{p}_1-\vec{q})^2}{2m}-\sum_{i=2}^{A}\frac{|\vec{p}_i|^2}{2m}\,i\epsilon}\,\Gamma\left(p_1-q;\{p_i\}\right) \tag{2.6}$$

The above calculation did not take into account the possible singularities in the nucleon amplitude, since their positions are determined by the mass of hadrons $\tilde{p}_{0,i} \approx m_\pi$. Closing the contour on these singularities, we obtain a smaller contribution of the order of $1/m_\pi R_A$.

Introducing the wave function of the nucleus as follows

$$\Psi\left(\{r_i\}\right) \;=\; \int\prod_{i=1}^{A}\frac{d^3p_i}{(2\pi)^3}e^{i\vec{p}_i\cdot\vec{r}_i}\,\Gamma\left(p_1;\{p_i\}\right)\,\frac{1}{A\,\varepsilon-\sum_{i=1}^{A}\frac{|\vec{p}_i|^2}{2m}-i\epsilon} \tag{2.7}$$

we can rewrite eq. (2.5) in the form

$$A_A\left(s,q^2;\text{figure } 1-a\right) \;=\; A_N\left(s,q_\perp^2\right)\;\int\prod_{i=1}^{A}d^3\,r_i\,e^{i\vec{q}_\perp\cdot\vec{r}_{1,\perp}}\,|\Psi\left(\{r_i\}\right)|^2 \tag{2.8}$$

$$\rightarrow\;A_N\left(s,q_\perp^2=0\right)\int\prod_{i=1}^{A}d^3\,r_i\,e^{i\vec{q}_\perp\cdot\vec{r}_{1,\perp}}\,|\Psi\left(\{r_i\}\right)|^2\;\equiv\;A_N\left(s,q_\perp^2=0\right)\,S\left(q_\perp^2\right)$$

which is eq. (2.1) in momentum representation. In deriving eq. (2.8), we used the fact that in $S\left(q_\perp^2\right)$, the typical $q_\perp \propto 1/R_A$, which is much smaller than the characteristic $q_\perp$ in the nucleon amplitude, and which can be considered to be a constant as far as the $q_\perp$ dependence is concerned. Now we want to show that the diagram of figure 1-b leads to the following contribution

$$A_A\left(s,b;\text{figure } 1-b\right) \;=\; i\frac{1}{2}\left(\int d^2b' A_N\left(s,b'\right)\right)^2 S^2\left(b\right) \tag{2.9}$$

It turns out that eq. (2.9) can be obtained with the additional assumption that the wave function can be factorized as

$$\Psi\left(\{r_i\}\right) \;=\; \prod_{i=1}^{A}\Psi\left(r_i\right)\;\text{, which gives }\; S\left(b\right)=\int dz\left|\Psi\left(b,z\right)\right|^2\;\text{, with } \vec{r}=(\vec{b}_\perp,z). \tag{2.10}$$

This means that there are no correlations between different nucleons in a nucleus. In other words, we describe the nucleus as the nucleons that are moving in the external potential in the spirit of the Hartree-Fock approach.

The amplitude for the diagram of figure 1-b has the form

$$\begin{aligned}A_A\left(s,q^2;\text{figure } 1-b\right)=&\int\frac{d^4k}{(2\pi)^4 i}\frac{1}{m_p^2-(P+k)^2}\int\frac{d^4p_1}{(2\pi)^4 i}\frac{d^4p_2}{(2\pi)^4 i}\prod_{i=3}^{A}\frac{d^4p_i}{(2\pi)^4 i}\Gamma\left(p_1,p_2,\{p_i\}\right)\\ &\times\frac{1}{m^2-p_2^2-i\epsilon}\frac{1}{m^2-p_i^2 1-i\epsilon}A_N\left(s,k_\perp^2;p_1^2,(p_1-k)^2\right);\\ &\times\frac{1}{m^2-(p_1-k)^2-i\epsilon}\;A_N\left(s,(q-k)_\perp^2;p_2^2,(p_2-q+k)^2\right)\\ &\times\frac{1}{m^2-(p_2-q+k)^2-i\epsilon}\frac{1}{m^2-p_i^2-i\epsilon}\Gamma(p_1-k,p_2-q+k,\{p_i\})\end{aligned} \tag{2.11}$$

We integrate first over the momentum k. Rewriting d^4k as $dk_0 d(k_0-k_z)d^2k$, and closing the contour of integration over the variable k_0-k_z, on the pole $(P+k)^2=m_p^2$, leads to a factor of $2\pi i/P_0$. For the integration over k_0, we can also close the contour on one of the poles: $(p_1-k)^2=m^2$ or $(p_2-q+k)^2=m^2$, which can be rewritten as $m\,\varepsilon+2m\left(\tilde{p}_{0,1}-k_0\right)-(\vec{p}_1-\vec{k})^2-i\epsilon=0$ and $m\,\varepsilon+2m\left(\tilde{p}_{0,2}-q_0+k_0\right)-(\vec{p}_2-\vec{q}+\vec{k})^2$. This integration brings an additional factor of $2\pi i/2m$. Therefore, the integration over k leads to the following contribution, namely $i\,d^2k/((2\pi)^2\,s)$. Evaluating all the integrations over $\tilde{p}_{0,i}$ in the same way as we did when calculating the diagram of figure 1-a, we reduce eq. (2.11) to the following expression

$$\begin{aligned}A_A\left(s,q_\perp^2;\text{figure } 1-b\right)=&\frac{i}{s}\int\frac{d^2k}{(2\pi)^2}\int\prod_{i=1}^{A}\frac{d^3p_i}{(2\pi)^3}\Gamma\left(p_1;\{p_i\}\right)\frac{1}{A\varepsilon-\sum_{i=1}^{A}\frac{|\vec{p}_i|^2}{2m}-i\epsilon}A_N\left(s,k_\perp^2;p_1^2\right)\\ &\times A_N\left(s,(q-k)_\perp^2;p_1^2\right)\frac{1}{A\varepsilon-\frac{(\vec{p}_1-\vec{k})^2}{2m}-\frac{(\vec{p}_2-\vec{q}+\vec{k})^2}{2m}-\sum_{i=3}^{A}\frac{|\vec{p}_i|^2}{2m}i\epsilon}\Gamma\left(p_1-q;\{p_i\}\right)\end{aligned} \tag{2.12}$$

Eq. (2.12) can be easily rewritten in coordinate representation, by introducing the wave function of eq. (2.7), namely

$$A_A\left(s,b;\text{figure } 1-b\right) \;=\; \frac{i}{s}A_N^2\left(s,q^2=0\right)\int dz_1\int^{z_1}dz_2\left|\Psi\left(,b,z_1;b,z_2;\{r_i\}\right)\right|^2 \tag{2.13}$$

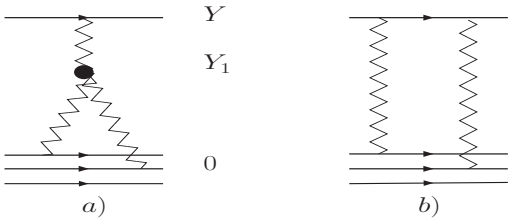

Figure 2: The first fan diagram (figure 2-a) for the interaction of Pomerons (reggeized gravitons) and the eikonal diagram (figure 2-b).

Using the non-relativistic normalization for the scattering amplitude ($A_{\rm nr} = A/s$)[2] and eq. (2.10), we can see that we obtain eq. (2.9). It should be noted that the factor 1/2 stems from the z_2 integration, which is not restricted in eq. (2.9), in contrast with eq. (2.13). All calculations above have been done to illustrate two points, namely that we do not need to assume that the nucleon amplitude should be pure imaginary, but we need to assume a very simple model for the nuclei.

Calculating the amplitude for the interaction with any number of nucleons in a nucleus, we obtain the simple formula for the nucleus scattering amplitude (see a more detailed derivation in ref. [21]), namely,

$$A_A(s,b) \;=\; i\left(1 \;-\; \exp\left(i\int db' A_N(s,b')\,S(b)\right)\right) \tag{2.14}$$

In deriving eq. (2.1) we considered the propagators of the projectile and the target (nucleons in a nucleus) in flat space but not in AdS_5. In the next section we will comment on this but the main argument is very simple: the trajectory of a fast moving particle can be replaced by the straight line in curved space as well as in flat one. The second assumption was that we considered in figure 1-b the projectile as the intermediate state.

Using the AdS/CFT correspondence we can estimate the accuracy of this (eikonal) approach in the N=4 SYM case. Indeed, at first sight we can expect from the AdS/CFT correspondence, that the main contribution will stem from the fan diagrams, the first of which is shown in figure 2-a, as it happens in this theory in the region of small coupling constant. In fact, from the region of small coupling we expect that (i) this diagram has the contribution of the order of $(\alpha_s^5/\Delta)\,s^{2\Delta}$, where $\Delta \propto \alpha_s$ is the intercept of the BFKL Pomeron; (ii) the typical value of $Y - Y_1 \approx 1/\Delta \gg 1$ and (iii) the value of this contribution is closely related to the process of diffractive dissociation of the projectile. Since $Y - Y_1 \gg 1$ it is reasonable to consider the exchange of the BFKL Pomeron. The eikonal diagram of figure 2-b has the same order of magnitude but it turns out (see ref. [25]) that this diagram is included in the diagram of figure 2-a in the region of integration $Y - Y_1 \approx 1$ where we cannot use Pomeron exchange. Therefore, in the weak coupling limit the full set of diagrams at high energy can be reduced to the "fan" diagrams. It is worth mentioning that in the weak coupling limit the eikonal diagram of figure 2-b has the same intermediate

[2]Starting from this equation we will use the notation A_N and A_A for the non-relativistically normalized amplitudes, hoping that it will not lead to any misunderstanding.

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state as the initial one (the colourless dipole) since it turns out that colourless dipoles are diagonalized by the interaction matrix (see ref. [29]).

In the strong coupling limit of N=4 SYM, due to the AdS/CFT correspondence, the strong interaction of Pomerons is replaced by the weak interaction of the reggeized gravitons, with intercepts $\Delta = 1-2/\sqrt{\lambda}$, and therefore in the triple Pomeron diagram the typical value of $Y - Y_1 \approx 1/(\Delta = 1 - 2/\sqrt{\lambda}) \approx 1$. It means that diffraction production, which can contribute and was neglected in the eikonal (Glauber-Gribov) approach , is the process in which low masses are produced. For $Y - Y_1 \approx 1$ there are no reasons to replace the amplitude by the reggeized graviton exchange. Using the AdS/CFT correspondence we expect that in the diagram of figure 2-b the same as the initial state is produced. On the other hand, the process of diffraction production of low mass can be easily taken into account in the eikonal approach, and does not change neither the energy nor the impact parameter dependence that has been discussed here. The cross section of the diffraction dissociation is proportional to the imaginary part of the reggeized graviton exchange which is small of the order of $2/\sqrt{\lambda}$. Therefore, at least within this accuracy ($2/\sqrt{\lambda}$), the exchange of two gravitons between the projectile and the target (eikonal diagram of figure 2-b) prevails.

2.2 Nucleon amplitude in N=4 SYM

The main contribution to the scattering amplitude at high energy in N=4 SYM, stems from the exchange of the graviton.[3] The formula for this exchange has been written in ref. [4, 6], (see also ref. [10] for its interpretation). In flat space this amplitude has the following form

$$A_g(s,q) \;\propto\; T_{\mu\nu}\,(p_1,p_2)\,G_{\mu\nu\mu'\nu'}\,(q)\;T_{\mu'\nu'}\,(p_1,p_2) \;\xrightarrow{s\gg q^2}\; s^2\,\frac{1}{q_\perp^2} \tag{2.15}$$

where $T_{\mu,\nu}$ is the energy-momentum tensor, and G is the propagator of the massless graviton. The last expression in eq. (2.15), stems from the fact that for high energies, $T_{\mu,\nu} = p_{1,\mu}p_{1,\nu}$, and $q^2 = -q_\perp^2$ (see the previous section). However, we are interested in N=4 SYM in a space with curvature, namely AdS_5. AdS_{d+1} corresponds to an hyperboloid in $d+2$ flat space, namely

$$-Y_{-1}^2 \,+\, Y_0^2 \,+\, \sum_{I=1}^{d} X_i^2 = -L^2 \tag{2.16}$$

with curvature $R = -d(d-1)L^2$. Introducing new coordinates

$$x_i \;=\; \frac{L\,X_i}{Y_0 + Y_{-1}}\,; \qquad z \;=\; \frac{L^2}{Y_0 + Y_{-1}}\,; \tag{2.17}$$

we reduce the introduced metric to the following form

$$ds^2 \;=\; \frac{L^2}{z^2}\left(dz^2 \,+\, \sum_{i=1}^{d} dx_i^2\right) \;=\; \frac{L^2}{z^2}\,\left(dz^2 \,+\, d\vec{x}^2\right) \tag{2.18}$$

[3] Actually, the graviton in this theory is reggeized [3], but it is easy to take this effect into account (see refs. [3, 5, 10]).

In the flat $d+2$ dimensional space, the scalar propagator is the following (with $Y_+ = Y_0 + Y_{-1}$ and $Y_- = Y_0 - Y_{-1}$)

$$\begin{aligned} G\left(X_i, Y_+, Y_-; X_i', Y_+', Y_-'\right) &= \int \prod_{i=1}^{d} \frac{dk_i}{2\pi} \frac{dp_+ dp_-}{(2\pi)^2} \frac{1}{\sum_{i=1}^{d} k_i^2 + p_+ p_-} e^{-i\vec{k}\cdot\vec{X} - i\frac{1}{2}p_+ Y_- - i\frac{1}{2}p_- Y_+} \\ &= \int_0^\infty dt \int \prod_{i=1}^{d} \frac{dk_i}{2\pi} \frac{dp_+ dp_-}{(2\pi)^2} \exp\left(-tk^2 - tp_+p_- - i\vec{k}\cdot\vec{X} - i\frac{1}{2}p_+ Y_- - i\frac{1}{2}p_- Y_+\right) \\ &= (2\pi)^{-d/2-1} \int_0^\infty dt t^{-d/2-1} e^{-u/t} \xrightarrow{t\to 1/\xi} (2\pi)^{-d/2-1} \int_0^\infty d\xi (\xi)^{d/2-1} e^{-\xi u} \\ &= (2\pi)^{-d/2-1} \Gamma\left(d/2\right) u^{-\frac{1}{2}d} \end{aligned} \tag{2.19}$$

In eq. (2.19), u is a new variable which is equal to

$$u \;=\; \frac{(z-z')^2 + (\vec{x}-\vec{x}')^2}{2\, z\, z'}. \tag{2.20}$$

In eq. (2.19), we re-write the integration measure of the momenta in $d+2$ dimensional space, namely $\prod_{i=1}^{d+2} dp_i$, as $\prod_{i=1}^{d} dk_i\, dp_+ dp_-$, where p_+ and p_- are the conjugated momenta to Y_- and Y_+, respectively.

However, the propagator of eq. (2.19) does not satisfy the correct boundary condition, for example, $G\left(X_i, Y_+, Y_-; X_i', Y_+', Y_-'\right)$ should approach $\delta\left(\vec{X} - \vec{X}'\right)$ as $z \to z'$, which is not the case for this equation. One of the reasons why this happens, is that we have to guarantee that $Y_+ > 0$.[4] The easiest way to impose such a condition, is to change eq. (2.19) to

$$\begin{aligned} G\left(X_i, Y_+, Y_-; X_i', Y_+', Y_-'\right) \;&=\; \int \prod_{i=1}^{d} \frac{dk_i}{2\pi} \frac{dp_+ dp_-}{(2\pi)^2} \frac{1}{\sum_{i=1}^{d} k_i^2 + p_+ p_-} \frac{1}{p_-} e^{-i\vec{k}\cdot\vec{X} - i\frac{1}{2}p_+ Y_- - i\frac{1}{2}p_- Y_+} \\ &=\; \int \prod_{i=1}^{d} \frac{dk_i}{2\pi} \frac{dp_+}{(2\pi)^2} \frac{1}{k^2} \left\{ e^{i\frac{k^2}{p_+} Y_+} \;-\; 1 \right\} e^{-i\vec{k}\cdot\vec{X} - i\frac{1}{2}p_+ Y_-} \end{aligned} \tag{2.21}$$

One can see from eq. (2.21) that $\left(\sum_{i=0}^{i=3} \partial^2/\partial^2 X_i + \partial^2/\partial^2 Y_0 - \partial^2/\partial^2 Y_{-1}\right)$ $G\left(X_i, Y_+, Y_-; X_i', Y_+', Y_-'\right)$ is defined only for $Y_+ > 0$. Therefore, the solution of the equation for the Green's function also will be determined only for $Y_+ > 0$.

Notice that the mass of the graviton is equal to zero even in the AdS_{d+1} space with curvature. Having this in mind, the easiest way to find the correct propagator, is to write the wave equation directly in the AdS_{d+1} space, assuming that the mass of the graviton is equal to zero, and that $G\left(X_i, Y_+, Y_-; X_i', Y_+', Y_-'\right)$ is a function of the variable u of eq. (2.20). The action for such a particle has the following form

$$S[\phi] = \frac{1}{2} \int d^d x\, dz\, \sqrt{g}\, g^{\mu,\nu}\, \partial_\mu \phi\, \partial_\nu \phi \tag{2.22}$$

[4] We thank Chung-I Tan for the fruitful discussion of all aspects of high energy scattering in N=4 SYM, in particular, the $Y_+ > 0$ condition.

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where the metric is given by eq. (2.18) . Using eq. (2.22) and eq. (2.18), it is easy to obtain the wave equation for $G\left(X_i, Y_+, Y_-; X_i', Y_+', Y_-'\right) = G(u)$. It has the form [22 – 24]

$$\frac{1}{\sqrt{g}}\,\partial_\mu \sqrt{g}\, g^{\mu.\nu}\partial_\nu\, G(u) = 0\,; \tag{2.23}$$

$$z^2\nabla_x^2 G(u) + z^{d+1}\frac{\partial}{\partial z}\left[z^{-d+1}\,\frac{\partial G(u)}{\partial z}\right] = 0\,; \tag{2.24}$$

$$u\,(u+2)\,G_{u,u}\,(u) + (d+1)\,G_u\,(u) = 0\,; \tag{2.25}$$

The solution to eq. (2.25), which satisfies all the necessary boundary conditions: $G(u) \xrightarrow{u\to\infty} 0$ and $G(u) \xrightarrow{z\to z'} \delta\,(\vec{x}-\vec{x}')$ has the form [22 – 24, 6]

$$G(u) = \frac{d-1}{2^{d+1}}\left(\frac{1}{4\pi}\right)^{\frac{1}{2}d}\left(-\frac{2}{u}\right)^d\,{}_2F_1\left(d, \frac{1}{2}(d+1), d+1, -\frac{2}{u}\right) \tag{2.26}$$

As has been discussed (see eq. (2.15)), we need an expression for the propagator of the graviton, which at high energy depends only on the transverse coordinates for the scattering. Therefore, we need $G(u)$ for AdS_{2+1}, which is equal to

$$G_3(u) = \frac{1}{4\pi}\,\frac{1}{\left\{1+u+\sqrt{u(u+2)}\right\}^2\,\sqrt{u(u+2)}} \tag{2.27}$$

with

$$u = \frac{(z-z')^2+b^2}{2\,z\,z'} \tag{2.28}$$

where b is the impact parameter for the scattering amplitude.

For the eikonal formula, we need to evaluate the integral over b, which can be easily done noticing that

$$d\left\{1+u+\sqrt{u(u+2)}\right\}/db^2 = \frac{1}{2\,z\,z'}\,\frac{\left\{1+u+\sqrt{u(u+2)}\right\}}{\sqrt{u(u+2)}} \tag{2.29}$$

The result is

$$\begin{aligned} G(z,z') &= \int d^2b\, G_3\,(u) \qquad (2.30)\\ &= \frac{z\,z'}{4}\,\frac{1}{\left\{1+u(b=0)+\sqrt{u(b=0)(u(b=0)+2)}\right\}^2} = z\,z'\,\frac{z^2\,z'^2}{(z^2+z'^2+|z^2-z'^2|)^2} \end{aligned}$$

This equation provides us with the factor which enters into eq. (2.15), instead of $1/q_\perp^2$. It turns out that in curved space we need to change [4]

$$s \;\to\; \tilde{s} = \frac{s}{\sqrt{g_{+-}(z)\,g_{-+}(z')}} = \frac{zz's}{R^2}. \tag{2.31}$$

For calculating the nucleon amplitude, we need to multiply eq. (2.30) by the coupling constant, and integrate over the nucleon wave function [4, 6]. Therefore, the nucleon amplitude is equal to

$$\int d^2b A_N(s,b) = ig_0^2 s \int dz' z z' G(z,z') |\Phi(z')|^2 \;=\; ig_0^2 s \int dz' |\Phi\left(z'\right)|^2 z'^2 \frac{z^2 z'^2}{(z^2+z'^2+|z^2-z'^2|)^2}$$
$$\xrightarrow{z \ll z'} \frac{ig_0^2 s}{4} z^4 \int dz' |\Phi\left(z'\right)|^2 = \frac{ig_0^2 N_c s}{4} z^4 \quad (2.32)$$

Here g_0^2 is the dimensionless constant, which is equal to $\kappa_5^2/2L^3$, where κ_5 is the five dimensional gravity. $g_0^2 \;\propto\; 1/N_c^2$ where N_c is the number of colours. We do not know anything about the nucleon wave function, except that the integral over z' converges, and it is proportional to N_c. Therefore, the amplitude is proportional to $A_N \propto s/N_c$ and it is small for $sz^2 < N_c$. It grows and becomes of the order of 1 due to the reggeization of the graviton. The graviton propagator in eq. (2.32) should be replaced by the propagator of the Pomeron, in the way as has been suggested in refs. [3, 10, 5]. This modification for our case is described in section 5. In eq. (2.32), we consider $\int dz' |\Phi z')^2 = N_c$.

As has been discussed, we use the propagator for a fast moving particle in the form

$$G\left(k_+, k_-; \vec{b}_1 - \vec{b}_2; z_1 - z_2\right) \;=\; \frac{1}{k_+(k_- \,-\, i\epsilon)}\, \delta^{(2)}\left(\vec{b}_1 - \vec{b}_2\right)\, \delta\left(z_1 - z_2\right) \quad (2.33)$$

Eq. (2.33) follows directly from eq. (2.21) . Indeed for large k_+ the pole in the integrant of eq. (2.21) is located at $k_- \;=\; (k_\perp^2 - p_+p_- - i\epsilon)/k_+ \;\to\; 0 - i\epsilon$. Therefore, $\sum_{i=1}^{d} k_i^2 + p_+p_-$ can be replaced by $k_+(k_- + i\epsilon)$. Substituting this expression in eq. (2.21) one can see that $G\left(k_+, k_-; \vec{b}_1 - \vec{b}_2; z_1 - z_2\right)$ has the form of eq. (2.33) with an additional factor $\Theta(z_1 + z_2)$ which is equal to 1.

Eq. (2.33) for $G\left(k_+, k_-; \vec{b}_1 - \vec{b}_2; z_1 - z_2\right)$ can be derived directly from eq. (2.23) and eq. (2.24). Indeed, going to Fourier transform for coordinates $x_i\,(i \;=\; 1, \ldots, d)$ and to Laplace transform for coordinate z we can rewrite eq. (2.24) in the form

$$k^2\, \tilde{G}'_p(\{k_i\}; p) \;-\; (d-1)\, p\, \tilde{G}\{k_i\}; p) \;-\left(p^2\, \tilde{G}(\{k_i\}; p)\right)'_p \;=\; 0 \quad (2.34)$$

The solution to this equation has the form

$$\tilde{G}(\{k_i\}; p) = \frac{1}{k^2 - p^2}\left(\frac{k^2}{k^2 - p^2}\right)^{\frac{d-1}{2}} = \frac{1}{k_+k_- - k_\perp^2 - p^2}\left(\frac{k_+k_- - k_\perp^2}{k_+k_- - k_\perp^2 - p^2}\right)^{\frac{d-1}{2}} \quad (2.35)$$

For large k_+ eq. (2.35) leads to

$$\tilde{G}(\{k_i\}; p) \;\xrightarrow{k_+ \ll \{k_\perp \text{ and } p\}}\; \frac{1}{k_+\,(k_- \;-\; i\,\epsilon)} \quad (2.36)$$

Eq. (2.36) gives eq. (2.33) which we use in our calculations.

2.3 Eikonal formula in N=4 SYM

Eq. (2.14) can be easily rewritten for the case of N=4 SYM in the following way using eq. (2.32)

$$A_A(s,b) \;=\; i\int d\,z\,|\Phi_p(z)|^2\,\left\{1\;-\;e^{i\,s\,\frac{g_0^2\,N_c}{4}\;z^4\,S(b)}\right\} \tag{2.37}$$

where Φ_p is the wave function of the projectile. This formula is almost the same as the eikonal formula for the hadron-nucleus interaction, except that the nucleon amplitude is purely real in our case.

The scattering amplitude at fixed z

$$A_A\,(s,b;z) \;=\; i\,\left\{1\;-\;e^{i\,s\,\frac{g_0^2\,N_c}{4}\;z^4 S(b)}\right\} \tag{2.38}$$

can be rewritten in the following way:

$$A_A\,(s,b;z) \;=\; \sin\left[s\,\frac{g_0^2\,N_c}{4}\;z^4\,S\,(b)\right]\;+\;i\,2\sin^2\left[s\,\frac{g_0^2\,N_c}{8}\;z^4\,S\,(b)\right] \tag{2.39}$$

One can see that the real and imaginary part of the amplitude are of the same order in contrast with the black disc behavior, for which only the imaginary part survives at high energy. One can see that the amplitude of eq. (2.39) satisfies the following unitarity constraint

$$2ImA_A\,(s,b;z) \;=\; |A_A\,(s,b;z)\,|^2 \tag{2.40}$$

Comparing eq. (2.40) with the general unitarity constraint, namely,

$$2ImA\,(s,b;z) \;=\; |A\,(s,b;z)\,|^2\;+\;G_{\rm inel}\,(s,b;z)$$

one can see that eq. (2.39) leads to only elastic scattering at high energy, in direct contradiction with our intuition based on the parton approach.

For the general formula of eq. (2.37), eq. (2.40) means that

$$\begin{aligned}\sigma_{\rm tot} \;&=\; 2\int d^2b\int d\,z\,|\Phi_p(z)|^2\,Re\left\{1\;-\;e^{i\,s\,\frac{g_0^2\,N_c}{4}\;z^4 S(b)}\right\}\;=\\ \sigma_{\rm diff}\;+\;\sigma_{\rm el}\;&=\;\int d^2b\int d\,z\,|\Phi_p(z)|^2\,\left|1\;-\;e^{i\,s\,\frac{g_0^2\,N_c}{4}\;z^4\;S(b)}\right|^2\end{aligned} \tag{2.41}$$

In other words, only the processes of diffractive dissociation contribute at high energy.

3. DIS with nuclei: general formulae

For calculating DIS, we need to specify the wave function of the projectile in eq. (2.39). In N=4 SYM, the natural probe for DIS is $\mathcal{R}$-current ($\mathcal{R}$-boson) [19], and the wave function for this probe satisfies eq. (2.24). However, in DIS we fix the virtuality of the probe (see figure 3). It means that in terms of eq. (2.19), $\sum_{i=1}^{d} k_i^2 = -Q^2$. Therefore, the wave

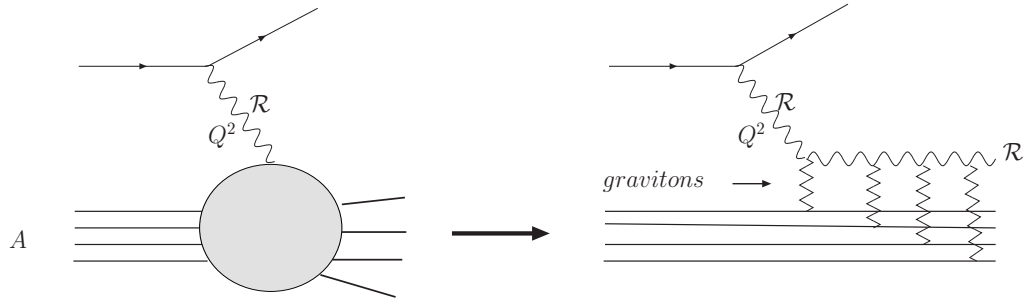

Figure 3: DIS with the nuclear target. The wave line denotes the $\mathcal{R}$ current ($\mathcal{R}$-boson), while the zigzag lines show the graviton exchanges. Q^2 is the virtuality of the probe.

function is described by eq. (2.24) with $d = 0$, but with $\nabla_x^2 \Psi = -Q^2 \Psi$, and the equation can be rewritten in the form [19]

$$-z^2\, Q^2 \Psi_{\mathcal{R}}\left(Q^2, z\right) + z\, \frac{d\Psi_{\mathcal{R}}\left(Q^2, z\right)}{dz} \;+\; z^2\, \frac{d^2\Psi_{\mathcal{R}}\left(Q^2, z\right)}{(dz)^2} \;=\; 0 \tag{3.1}$$

The solution to eq. (3.1) is

$$\Psi_{\mathcal{R}}\left(Q^2, z\right) \;=\; Q\, z K_1\left(Q\, z\right) \tag{3.2}$$

However, $\mathcal{R}$ - boson is a vector with $d+1$ components. The careful analysis of ref. [19] shows that eq. (3.2) describes only d components of this vector, while the $(d+1)$-th component has a different dependence on Qz. Finally [10],

$$|\Psi\left(Q^2, z\right)|^2 \;=\; \left(K_1^2\left(Q\, z\right) \;+\; K_0^2\left(Q\, z\right)\right)\, z^3 \tag{3.3}$$

The deep inelastic structure function has the following form [19, 10]

$$F_2\left(Q^2, x = Q^2/s\right) \;= \tag{3.4}$$
$$= C\alpha' Q^6 \int d^2b \int dz z^3 \left(K_1^2\left(Qz\right) + K_0^2\left(Qz\right)\right) 2Re\left\{1 - \exp\left(i\frac{g_0^2 N_c}{4}\frac{Q^2}{x} z^4 S\left(b\right)\right)\right\}$$

where C is a dimensionless constant.

Changing the variable z to $\zeta = Q\, z$, one can see that F_2 can be written in the form

$$F_2\left(Q^2, x = Q^2/s\right) \;=\; C\; Q^2 \int d^2b\, \Phi\left(\tau(Q, x, b)\right) \;= \tag{3.5}$$
$$= C\; Q^2 \int d^2b \int d\zeta\, \zeta^3 \left(K_1^2\left(\zeta\right) \;+\; K_0^2\left(\zeta\right)\right) \;\; Re\left\{1 \;-\; \exp\left(i\,\frac{1}{\tau}\,\zeta^4\right)\right\}$$

where

$$\tau \;=\; \frac{Q^2\, x}{\frac{g_0^2 N_c}{4}\, S\left(b\right)} \;=\; \frac{Q^2}{Q_s^2} \tag{3.6}$$

One can see that the DIS structure function shows the geometrical scaling behavior with the saturation momentum, which we can find from the equation with $\tau = 1$. It is equal to

$$Q_s^2\left(x\right) \;=\; g_0^2\, N_c\, S\left(b\right)/(4\, x) \;\propto\; \frac{A^{\frac{1}{3}}}{N_c}\,\frac{1}{x} \tag{3.7}$$

Therefore, F_2 shows the same main features as F_2 in high density QCD [7–9, 26], namely the geometrical scaling behavior, large values of the saturation scale in the region of low x, and the expected dependence of $Q_s^2 \propto A^{1/3}$. Actually, our analysis of Q_s repeats the one in ref. [10], and the difference between them stems from our integration over the impact parameters.

One can see from figure 4 that the function Φ has the same behavior as we expected from high density QCD, namely it approaches unity at small values of τ. Such a behavior looks strange, especially if we compare this function with eq. (2.39), which leads to an amplitude that oscillates between 0 and 2. Let us consider $\tau > 1$. In this case, we can replace the modified Bessel functions (McDonald functions) in eq. (3.5) by their asymptotic expression, namely, $K_n(\zeta) \to \sqrt{2\pi/\zeta}\,\exp(-\zeta)$, and in this case eq. (3.5) has the form

$$\begin{aligned} F_2\left(Q^2, x = Q^2/s\right) &= C\,Q^2 \int d^2b\,\Phi\left(\tau(Q,x,b)\right) = \\ &= C\,2\,\pi\,Q^2 \int d^2b \int d\zeta\,\zeta^2\,e^{-\zeta}\;\; Re\left\{1 \;-\; \exp\left(i\,\frac{1}{\tau}\,\zeta^4\right)\right\} \end{aligned} \tag{3.8}$$

The second term in $\{\dots\}$ can be estimated by the saddle point method. One can see that the saddle point value for $\zeta = \zeta_{\rm SP} = (-i\tau/3)^{1/3}$, and the integral has the following form

$$\Phi_{\rm SP}(\tau) \,=\, 1 \,-\, \sqrt{\frac{\pi\,\tau}{12 z^2_{\rm SP}}}\, z^2_{\rm SP}\, e^{-(2/3)(i\tau/3)^{1/3}} \;\to 1 \tag{3.9}$$

One can see that at $\tau \to 0$, the exponent $e^{-(2/3)(i\tau/3)^{1/3}} \to 1$, but the pre-exponential factor $\propto \tau^{5/6}$ vanishes. However, since $\zeta_{\rm SP} \ll 1$, at small values of τ we have to use the expression for the modified Bessel function at $\zeta \to 0$, namely $K_n(\zeta) \xrightarrow{\zeta \to 0} 1/\zeta^n$. Doing this, one can see that $\zeta \sim \tau^{1/4}$ contributes to the integral leading to the behavior of the second term in eq. (3.9) proportional to $\sqrt{\tau}$.

The above discussion shows that predictions of high density QCD differ from those of N=4 SYM, only in the way that $\Phi(\tau)$ approaches unity, namely $\Phi - 1 \propto \exp\left(-C\ln^2(1/\tau)\right)$ in high density QCD, and $\Phi - 1 \propto \exp\left(-\frac{1}{2}\ln(1/\tau)\right)$, in our approach.

We need to integrate $\Phi\left(\tau(b)\right)$ over b (see eq. (3.5)), to obtain the total cross section for DIS

$$\begin{aligned} \sigma_{\rm tot}\left(DIS\right) &= \frac{4\pi^2}{Q^2}\,F_2\left(Q^2, x = Q^2/s\right) \\ &= C \int d^2b\,\Phi\left(\tau(b)\right) \,=\, 2\pi\,C \int_{\tau(b=0)}^{\infty} \frac{d\tau}{\tau}\,\frac{S\left(b(\tau)\right)}{S_{b^2}\left(b(\tau)\right)}\,\Phi\left(\tau\right) \\ &\xrightarrow{x\to 0} C\,\pi R_A^2 \int_{\tau(b=0)\leq \tau_{\rm max}}^{\infty} d\tau\;\frac{\Phi\left(\tau\right)}{\tau}\;R\left(\tau\right) \;\text{ where }\; R\left(\tau\right) \,=\, \frac{S\left(b(\tau)\right)}{S_{b^2}\left(b(\tau)\right)} \end{aligned} \tag{3.10}$$

where $\tau = \tau_{\rm max}$ is the position of the maximum of the function $\Phi(\tau)$. The explicit form of the function $R(\tau)$ depends on the dependence of $S(b)$ on the impact parameter. We list

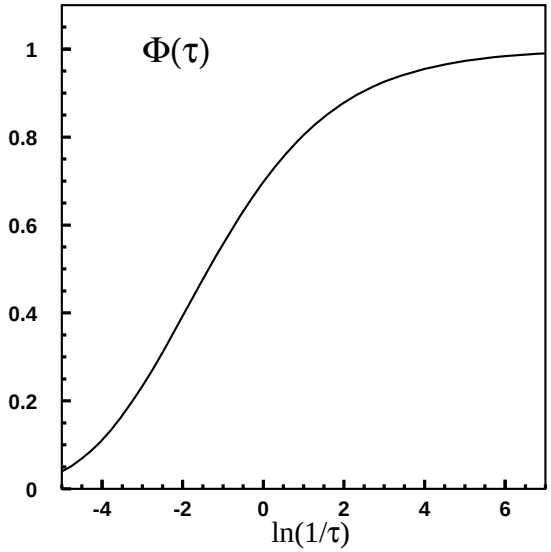

Figure 4: The τ dependence of function Φ.

below this function for several nucleus models:

$$R(\tau) = \begin{cases} \tau & \text{cylindrical nucleus} \quad S(b) = (A/\pi R_A^2)\,\Theta(R_A - b)\,; \\ 1 & \text{Gaussian form} \quad S(b) = (A/\pi R_A^2)\,\exp\left(-b^2/R_A^2\right)\,; \\ \tau(b=0)/\tau^3 & \text{spherical drop nucleus} \quad S(b) = (3A/4\pi^2\,R_A^2)\,\sqrt{R_A^2 - b^2}; \end{cases} \tag{3.11}$$

Unfortunately, in a realistic model of the nucleus with the Wood-Saxon form for the b dependence, we cannot give a simple analytical form of the function $R(\tau)$. In figure 5 we plot the integral over τ in eq. (3.10), for Gaussian b distribution. This distribution, being oversimplified, leads to correct estimates for the average characteristics of nuclei.

From eq. (3.10), one can see that the total cross section for DIS will be $2\pi R_A^2 \times \ln(1/\tau(b = 0))$, once more in accordance with our expectation from high density QCD for such $S(b)$. In the case of the Wood-Saxon parameterization, $S(b) \xrightarrow{b>R_A} \exp(-b/h)$ which leads to $\sigma_{\rm tot} \propto \ln^2(\tau(b=0)$. This behavior coincides with the expectation of high density QCD.

Therefore, the Glauber-Gribov approach leads to a behavior of the DIS structure function, which fully supports the high density QCD picture, reproducing the geometrical scaling behavior, and the existence of only one new scale, namely the saturation momentum.

The main difference between N=4 SYM and high density QCD, lies only in the relation between the total cross section and the cross section of diffractive dissociation. That is, $\sigma_{\rm tot}(DIS) = \sigma_{\rm diff}(DIS)$ for N=4 SYM, and $\sigma_{\rm tot}(DIS) \neq \sigma_{\rm diff}(DIS)$ but $\sigma_{\rm diff} \xrightarrow{x\to 0} \frac{1}{2}\sigma_{\rm tot}$ for high density QCD. In N=4 SYM, this equality means that the elastic cross section is equal to zero, in sharp contradiction with QCD and any parton interpretation of high

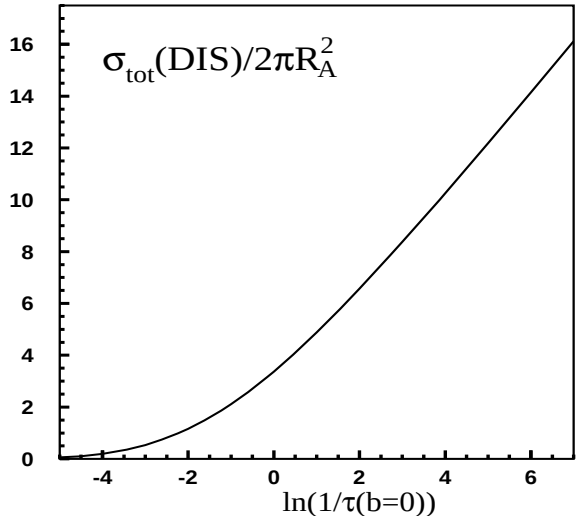

Figure 5: The integral over τ in eq. (3.10) as a function of $\tau(b=0)$ for Gaussian dependence of the nucleon density in nucleus versus the impact parameter.

energy scattering. However, this is a direct consequence of the fact that the graviton has spin 2. Actually, it has been shown in ref. [3] that its spin in N-4 SYM is not exactly 2, but rather $j_{\rm graviton} \equiv j_0 = 2 - 2/\sqrt{\lambda}$. Because of this, the amplitude of the interaction with the nucleon is not purely real, as it is given by eq. (2.32), but it has an imaginary part which is proportional to $2 - j_0$. figure 6 illustrates how this imaginary part influences the total and inelastic cross sections. We introduce the functions $\Phi_{\rm tot}$ and $\Phi_{\rm in}$ as

$$\sigma_{\rm tot} = \int d^2b\ \Phi_{\rm tot}(\tau) \ \text{ and } \ \sigma_{\rm in} = \int d^2b\ \Phi_{\rm in}(\tau)$$

The functions $\Phi_{\rm tot}$ and $\Phi_{\rm in}$ are shown in figure 6, for the imaginary part of the graviton exchange, which is 10% of the real part of the amplitude. One can see that such a small imaginary part generates a large inelastic cross section, and therefore the DIS structure function in N=4 SYM, with reggeized graviton, leads to a qualitative picture which is very difficult to differentiate from the high density QCD predictions.

To complete the proof of eq. (3.8), we need to discuss the contributions from multi-graviton exchanges in the nucleon amplitude. At first sight, they should be essential, since each graviton exchange brings in a factor (see eq. (2.32))

$$A_N^G(s,b) = ig_0 s \int dz' |\Phi(z')|^2 z z' G_3(u) \xrightarrow{b \gg z' > z} 8 i g_0 s z^4 \int dz' |\Phi(z')|^2 z'^4 / \left(b^2\right)^3 \qquad (3.12)$$

From eq. (3.12), one can see that the amplitude $A_N^G(s,b) \gg 1$ for $b^2 = b_0^2 \propto \left(is\, z^4\right)^{1/3}$. This means that we need to take into account all terms of the order of $\left(A_N^G\right)^n$. Using the eikonal formula for summing such terms, we see that for the nucleon amplitude we have

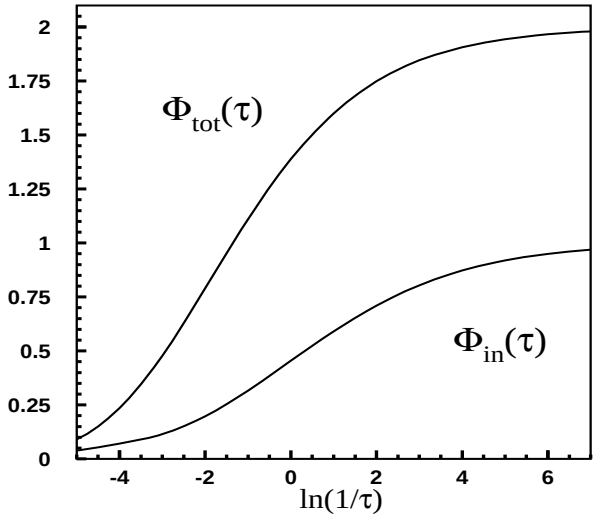

Figure 6: The behavior of the total and inelastic cross sections for the graviton exchange with 10% imaginary part of the amplitude.

the following expression, instead of the simple formula of eq. (2.32),

$$\int d^2b A_N(s,b) = \tag{3.13}$$
$$= \int d^2b \left\{1-\exp\left(A_N^G(s,b)\right)\right\} = \int d^2b \left\{1-\exp\left(ig_0 s \int dz' |\Phi(z')|^2 z z' G_3(u)\right)\right\}$$

The integral over b can be estimated as $\int d^2\, b\, A_N(s,b) \propto C\,\pi b_0^2 \propto \left(i\,s\,z^4\right)^{1/3}$. Using eq. (3.13), we can rewrite eq. (2.37) in the form

$$A_A(s,b) \;=\; i \int d\,z\,|\Phi_p(z)|^2 \left\{1 \;-\; e^{i\,C'\,\pi b_0^2\,S(b)}\right\} \tag{3.14}$$

For DIS we have

$$F_2\left(Q^2, x=Q^2/s\right) = \tag{3.15}$$
$$= C\alpha' Q^6 \int d^2b \int dz z^3 \left(K_1^2(Qz)+K_0^2(Qz)\right) 2Re\left\{1-\exp\left(i\pi C' b_0^2 z^4 S(b)\right)\right\}$$

where C and C' are dimensionless constants, whose values are irrelevant for our discussion.

Performing the integral over z using the asymptotic behavior of modified Bessel functions and the saddle point approach, one can see that the saddle point value of $z = z_{\rm SP}$ is equal to

$$z_{\rm SP} = \frac{Q^3}{i\,s\,S^3(b)} \;\approx\; \frac{Q^2}{i\,A\,s} \tag{3.16}$$

where A is the number of the nucleons in a nucleus. The value of F_2 in the saddle point is

$$F_2 \;\propto\; \exp\left(-i\,const\,Q^4/(sA)\right) \tag{3.17}$$

This formula, if correct, leads to a saturation scale $Q_s^2 \propto A/x$, in drastic contradiction with the prediction of high density QCD, both in the A and s dependencies. However, if we come back to eq. (3.12), we obtain

$$A_N^G(s,b) = 8ig_0 s z_{\rm SP}^4 \int dz' |\Phi(z')|^2 z'^4 / \left(b^2\right)^3 \;\propto\; is\left(\frac{Q^3}{As}\right)^4 \;\ll\; 1 \text{ for } s \gg Q^2(x\to 0) \quad (3.18)$$

From these estimates we conclude that the multi-graviton exchange does not contribute to DIS with a nuclear target, at low x.

4. DIS with nuclei: ultra high energy limit

The result of the previous section is, however, valid only for a limited range of energy. Indeed, we observe that the value of the typical impact parameters in the nucleon scattering amplitude ($b^2 = b_0^2 \propto \left(is\,z^4\right)^{1/3}$), grows with energy, and for energies larger than the energy ($s = s_{\rm crit}$) when $b_0 \geq R_A$, we cannot use the eikonal formula in the form of eq. (2.14). Indeed, for such large energies, the main assumption of the Glauber-Gribov approach does not work. This assumption has been discussed in eq. (2.8), which can be rewritten in the following way in the case of one graviton exchange

$$A_A(s,b) \;=\; \int d^2b'\, A_N\left(s,b'\right)\, S\left(\vec{b}-\vec{b}'\right) \;\to\; \int d^2b'\, A_N\left(s,b'\right)\, S(b) \qquad (4.1)$$

In eq. (4.1), we assume that in the interaction with one nucleon, the typical impact parameters are much smaller than R_A, which gives the scale for the impact parameter distribution in the nuclei. If the typical b in the nucleon interaction is larger than R_A, we have to use a different approximation, namely we need to rewrite eq. (4.1) in the form

$$A_A(s,b) \;=\; \int d^2b'\, A_N\left(s,b'\right)\, S\left(\vec{b}-\vec{b}'\right) \;\to\; A_N(s,b)\int d^2b'\, S(b) \;=\; A\; A_N(s,b) \quad (4.2)$$

This equation leads to a new formula for the scattering amplitude with a nucleus, instead of eq. (2.14), namely,

$$A_A(s,b) \;=\; i\,(\,1 \;-\; \exp\left(i\,A\,A_N(s,b)\right)) \qquad (4.3)$$

which leads to an expression for the DIS structure function in the form

$$\begin{aligned} F_2\left(Q^2, x{=}Q^2/s\right) \;&= \qquad\qquad (4.4)\\ &= C\alpha' Q^6 \int d^2b \int dz z^3 \left(K_1^2(Qz){+}K_0^2(Qz)\right) 2Re\left\{1{-}\exp\left(iAA_N(s,b)\right)\right\} \end{aligned}$$

where $A_N(s,b)$ is given by eq. (3.12). A_N can be rewritten at large b, using eq. (2.3) and eq. (3.12), in the form

$$A_N(s,b;z) \;=\; i\,\sin\left[s\,\frac{g_0^2\,N_c}{8}\;z^4/(b^2)^3\right] \;+\; 2\sin^2\left[s\,\frac{g_0^2\,N_c}{16}\;z^4/(b^2)^3\right] \qquad (4.5)$$

Substituting eq. (4.5), we do the integral over z using the steepest decent method. The most important part of the nucleon amplitude is the imaginary part, which leads to a damping of the interaction matrix (S-matrix) at high energies, provided the amplitude tends to unity. The saddle point for z is equal to

$$z_{\rm SP} \;=\; b^2 \left(\frac{Q}{4As \cos\left[\frac{g_0^2 N_c}{8}\; z^4/(b^2)^3\right]} \right)^{\frac{1}{3}} \tag{4.6}$$

Taking the integral using the steepest decent method we obtain

$$F_2\left(Q^2, x = Q^2/s\right) \;\propto\; Q^5 \int d^2b\, z_{\rm SP}^2 \frac{\sqrt{\pi}}{2\, z_{\rm SP}\sqrt{A\, s}} \exp\left(-5/4\; b^2 \;\left(-\frac{Q^4}{4A\, s} \right)^{\frac{1}{3}} \right) \tag{4.7}$$

where we replaced sines and cosines in eq. (4.5) and eq. (4.6), by unity since these functions cannot change the energy and Q dependence of the resulting amplitude.

From eq. (4.7), one can see that the typical values of the impact parameters are large and equal to

$$b_0^2 \;=\; 4/(5Qz_{\rm SP}) \;=\; \frac{4}{5}\left(-\frac{4As}{Q^4} \right)^{\frac{1}{3}} \;\gg\; z_{\rm SP}^2 \tag{4.8}$$

The resulting answer for F_2 is the following

$$F_2 \;=\; \propto\; \alpha' Q^2 \left(\frac{A\, s}{Q^2} \right)^{\frac{1}{3}} \;=\; \alpha' Q^2 A^{\frac{1}{3}} x^{-\frac{1}{3}} \tag{4.9}$$

Therefore, we see that we expect a very strange behavior from the point of view of high density QCD, both as function of A and x. The origin is clear. N=4 SYM has a massless particle, namely the graviton, and because of this the nucleon amplitude falls at large $b^2 \gg z^2 + z'^2$, as a power of $1/(b^2)^3$. Such a power-like behavior leads to a typical b which grows as a power of energy, (see ref. [30] for details), as has been demonstrated above. However, as has been shown in refs. [3, 5], actually the graviton has a mass which is not equal to zero if we dealing with the propagation of the graviton in AdS_5. This mass leads to a reggeization of the graviton, which has spin $j_0 = 2 - 2/\sqrt{\lambda} < 2$, in the scattering kinematic region where the square of the momentum transferred t is negative ($t < 0$). The fact that there is no massless particle in the curved space means that at large b, the amplitude should falls exponentially leading to a log energy dependence of the cross section. This is the reason why in the next section we will discuss the exchange of the reggeized graviton, and the Glauber- type formula which such an interaction induces.

5. DIS with nuclei: graviton reggeization

As has been discussed in ref. [5], for the exchange of the reggeized graviton we need to replace eq. (2.32) by a more general expression, namely

$$\int d^2\, b\, A_N\left(s, b\right) \;=\; i\, g_0^2 \frac{1}{\tilde{s}} \left\{ -\int \frac{d\, j}{2\pi\, i} \left(\frac{\tilde{s}^j \,+\, (-\tilde{s})^j}{\sin\, \pi j} \right) \int d^2b\; G_3\left(u, j\right) \right\} \tag{5.1}$$

where

$$G_3(u,j) = \frac{1}{4\pi}\frac{\left(1+u+\sqrt{u(u+2)}\right)^{2-\Delta_+(j)}}{\sqrt{u(u+2)}} \qquad (5.2)$$
$$\text{with } \Delta_+(j) = 2+\sqrt{4+2\sqrt{\lambda}(j-2)} = 2+\sqrt{2\sqrt{\lambda}(j-j_0)}$$

Using the definition of u given in eq. (2.28) and eq. (2.29), we can easily evaluate the integral over b in eq. (5.1) with the following result

$$\int d^2b G_3(u,j) = zz'\frac{1}{2-\Delta_+(j)}\left(1+u(b=0)+\sqrt{u(b=0)(u(b=0)+2)}\right)^{2-\Delta_+(j)} \qquad (5.3)$$

The integral over j in eq. (5.1) is a contour integral, and the contour is located to the right of all singularities of $\int d^2b\, G_3(u,j)$, but to the left of $j=2$, and the contour can be drawn to be parallel to the imaginary axis. In eq. (5.3), one can see that our singularity in j stems from the zero of the factor $2-\Delta_+(j)$. Denoting $\sqrt{2\sqrt{\lambda}|j-j_0|}=\nu$, we can rewrite the contribution of the square root singularity at $j=j_0$ in the following way

$$\begin{aligned}\int d^2bA_N(s,b) &= g_0^2 2zz'\left(\cot\frac{\pi j_0}{2}+i\right)\tilde{s}^{j_0-1} \qquad (5.4)\\ &\times\int_{i\epsilon-\infty}^{i\epsilon+\infty}\frac{d\nu}{\sqrt{\lambda}\pi}\exp\Big(-\nu^2/(2\sqrt{\lambda})\\ &\qquad +i\nu\ln\left\{1+u(b=0)+\sqrt{u(b=0)(u(b=0)+2)}\right\}\Big)\\ &\xrightarrow{z>z'} g_0^2 2zz'\left(\cot\frac{\pi j_0}{2}+i\right)\tilde{s}^{j_0-1}\int_{i\epsilon-\infty}^{i\epsilon+\infty}\frac{d\nu}{\sqrt{\lambda}\pi}\exp\left(-\nu^2/(2\sqrt{\lambda})+i\nu\ln\left(\frac{z}{z'}\right)\right)\end{aligned}$$

In the course of deriving eq. (5.4), we neglected in the signature factor the contribution of the term $\nu^2/(2\sqrt{\lambda})$, considering it to be smaller than j_0 ($j_0 \gg \nu^2/(2\sqrt{\lambda})$. The integral in eq. (5.4) can be evaluated such that it reduces to the following expression

$$\int d^2bA_N(s,b) = g_0^2 2zz'\left(\cot\frac{\pi j_0}{2}+i\right)\tilde{s}^{j_0-1}\sqrt{\frac{2}{\pi\sqrt{\lambda}\ln\tilde{s}}}\exp\left(-\frac{\sqrt{\lambda}\ln^2(z'/z)}{2\ln\tilde{s}}\right) \qquad (5.5)$$

The result of eq. (5.5) is obtained assuming that λ is fixed, but $s\to\infty$. From eq. (5.1), eq. (5.2) and eq. (5.3), we can recover a different limit, namely $\lambda\to\infty$ when $\tilde{s}\gg 1$. Indeed, in this limit $\Delta_+ \to 4+(\sqrt{\lambda}/2)(j-2)$. Since $2-\Delta_+(j)\to(\sqrt{\lambda}/2)(j-2)$, we can close the contour on the pole which stems from $2-\Delta_+(j)=0$. The signature factor can be rewritten in the form

$$\left(\cot\frac{\pi j_0}{2}+i\right)\xrightarrow{\lambda\to\infty}\frac{\sqrt{\lambda}}{\pi} \qquad (5.6)$$

Collecting everything together we obtain

$$\begin{aligned}\int d^2\,b\,A_N(s,b) &= g_0^2\,2\,z\,z'\left\{1-i\frac{2}{\sqrt{\lambda}}\right\}\int d^2\,b\,G_3(u,2)\\ &\equiv g_0^2\,2\,z\,z'\left\{1-i\frac{2}{\sqrt{\lambda}}\right\}\int d^2\,b\,G_3(u) \qquad (5.7)\end{aligned}$$

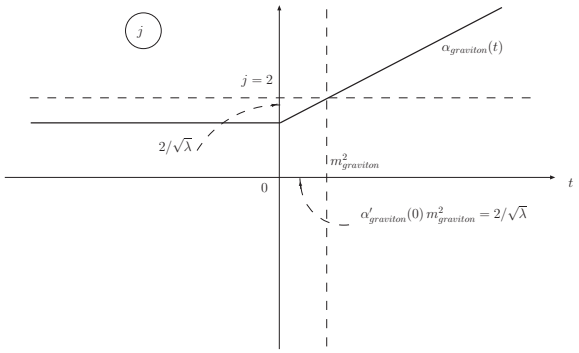

Figure 7: The graviton(Pomeron) trajectory in N=4 SYM as it follows from ref. [3].

We have used eq. (5.7) in our estimates of the value of the imaginary part of the nucleon scattering amplitude. eq. (5.7) leads to the exchange of the graviton with a small imaginary part, and this case has been considered in detail in this paper.

We concentrate our efforts on the limit $\tilde{s} \to \infty$,$\lambda = Const$. For this region we need to use eq. (5.4) for the nucleon amplitude. However, even more important for the high energy behavior of the amplitude, is the fact that the graviton has a mass in curved space (see figure 7). Therefore, the graviton trajectory which gives the dependence of the spin of the graviton on its mass, has the intercept $j_0 = \alpha_{\rm graviton}(0) = 2 - 2/\sqrt{\lambda}$, and the mass of the graviton is equal to $m^2_{\rm graviton} 2/(\sqrt{\lambda}\,\alpha'_{\rm graviton})$.[5] Therefore, in N=4 SYM all particles have masses, and the graviton is the lightest one. In such a theory, the large b dependence is determined by the mass of the lightest particle [31], namely $A_N(s,b) \to \exp\left(-m_{\rm graviton}\, b\right)$. This fact changes completely the ultra high energy behavior, that has been considered in the previous section. Assuming that the graviton mass is small, we can distinguish four different kinematic ranges of energy in the case if $R_A < 1/m_{\rm graviton}$; $z^2\, g_0^2 s \leq 1$; $z^2 g_0^2 s \geq 1$, but $b_0^2 \propto z^2 s \leq R_A^2$; and $R_A^2 \leq b_0^2 \propto z^2 s \leq 1/m^2_{\rm graviton}$ and $b_0^2 \propto z^2 s \geq 1/m^2_{\rm graviton}$. Nevertheless, we believe that the mass of the graviton should be such that $R_A \gg 1/m_{\rm graviton}$ if N=4 SYM pretends to describe the main features of the strong interaction. Indeed, we know experimentally that the lightest hadron is the π meson, and the large b dependence of the amplitude is proportional to $\exp\left(-b/2m_\pi\right)$. For a massive graviton the amplitude falls as $\exp\left(-b/m_{\rm graviton}\right)$. Therefore, to avoid contradiction with experiment, we need to assume that $m_{\rm graviton} > 2m_\pi$. Having this in mind, we will consider a modification to our formulae of the previous sections for the Glauber - Gribov approach in the case of the reggeized graviton, in three kinematic regions, which are $z^2\, g_0^2\, s \leq 1$, where we can restrict ourselves to the exchange of one graviton in the nucleon scattering amplitude; $z^2\, g_0^2\, s \geq 1$ but $b_0^2 \propto z^2 s \leq 1/m_{\rm graviton} \leq R_A^2$ (in this region the multi graviton exchange could be essential); and the asymptotic region where $z^2\, g_0^2\, s \geq 1$ but $b_0^2 \propto z^2\, s \geq 1/m_{\rm graviton} \leq R_A$. Of course, we can consider the kinematic region where $(1/m_{\rm graviton} \ln s) \geq R_A$, but in this region nuclei behave in the same way as the nucleons, and we are not interested in this region.

[5] In the previous sections we called $\alpha'_{\rm graviton}$ just α'.

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5.1 $z^2\, g_0^2\, s \;\leq\; 1$

In this kinematic region we can restrict ourselves to one reggeized graviton (Pomeron) exchange, and use eq. (5.5) instead of eq. (2.32). Therefore, we have

$$F_2\left(Q^2, x = Q^2/s\right) = C\alpha' Q^6 \int d^2b \int dz z^3 \left(K_1^2\left(Qz\right) + K_0^2\left(Qz\right)\right) 2Re\left\{1 - \right. \tag{5.8}$$
$$\left. - \exp\left(i g_0^2 N_c \int dz' |\Phi(z')|^2 2zz' \left(\cot\frac{\pi j_0}{2} + i\right) \tilde{s}^{j_0 - 1} \sqrt{\frac{2}{\pi\sqrt{\lambda}\ln\tilde{s}}} \exp\left(-\frac{\sqrt{\lambda}\ln^2(z'/z)}{2\ln\tilde{s}}\right) S\left(b\right)\right)\right\}$$

Two features of eq. (5.8) are quite different from eq. (3.4), namely that the nucleon amplitude has an imaginary part and shows a different z dependence. Roughly speaking, in eq. (5.8), $A_N \propto z^2$ instead of $A_N \propto z^4$ in eq. (3.4). The integral over z in eq. (5.8) can be evaluated using the steepest descent method. Using the asymptotic expression for the modified Bessel function, we reduce eq. (5.8) to the following expression

$$F_2\left(Q^2, x = Q^2/s\right) \;= \tag{5.9}$$
$$C\alpha' Q^5 \int d^2b \int z^2 dz e^{-Qz} Re\left\{1 - \exp\left(i g_0^2 N_c \xi(j_0) s^{j_0 - 1} (zz')^{j_0} S\left(b\right) E\left(\ln(z'.z)\right)\right)\right\}$$
$$\text{where} \quad E\left(\ln(z'.z)\right) \;=\; \sqrt{\frac{2}{\pi\,\sqrt{\lambda}\;\ln\tilde{s}}}\, \exp\left(-\frac{\sqrt{\lambda}\,\ln^2(z'/z)}{2\ln\tilde{s}}\right) \quad \text{and} \quad \xi(j) = i\,\cot\frac{\pi j_0}{2} \,-\, 1$$

Actually in eq. (5.9), we need to integrate over z', but we assume that the typical $z' \approx 1/\Lambda$, where Λ is a scale of hadrons (glueballs) in N=4 SYM, and we can replace it with some average value.

It is convenient to introduce new dimensionless variables $\hat{Q} = Q\, z'$, $\hat{s} = s z'^2$,$\hat{S}\left(b\right) = z'^2 S\left(b\right)$, $\hat{z} = z/z'$, for which the equation for the saddle point reads as follows

$$\hat{Q} \;=\; g_0^2\, N_c\, \xi(j_0)\, \hat{S}\left(b\right)\, \hat{s}^{j_0 - 1}\, \hat{z}_{\rm SP}^{j_0 - 1} \left(j_0 \,-\, \frac{\sqrt{\lambda}\,\ln(1/\hat{z}_{\rm SP})}{\ln\hat{s} + \ln\hat{z}_{\rm SP}}\right) E\left(\ln(1/\hat{z}_{\rm SP})\right) \tag{5.10}$$

Rewriting eq. (5.10) in the form

$$\ln\left(\frac{\hat{Q}}{\hat{S}\left(b\right)}\right) = \left(j_0 - 1\right) t \,-\, \frac{\sqrt{\lambda}\,(t - \ln\hat{s})^2}{2\,t} \,+\, w\left(\hat{z}_{\rm SP}\right) \tag{5.11}$$

where w is a smooth function of $\hat{z}_{\rm SP}$, and $t = \ln\left(\hat{s}\, z_{\rm SP}\right)$. The approximate solution for t is

$$\begin{aligned} t^{\pm} \;&=\; \ln\hat{s} - \frac{1}{\sqrt{\lambda}} \ln\left(\frac{\hat{Q}}{\hat{S}\left(b\right)} e^{-w(0)}\right) \,\pm\, \sqrt{\frac{-1 + j_0}{\sqrt{\lambda}}}\, \ln\hat{s}\,; \\ \hat{z}_{\rm SP}^{\pm} \;&=\; \exp\left(-\frac{1}{\sqrt{\lambda}} \ln\left(\frac{\hat{Q}}{\hat{S}\left(b\right)} e^{-w(0)}\right) \,\pm\, \sqrt{\frac{-1 + j_0}{\sqrt{\lambda}}}\, \ln\hat{s}\right)\,; \end{aligned} \tag{5.12}$$

Using eq. (5.12), we find that the DIS structure function is proportional to

$$F_2\left(Q^2,x\right) \propto Q^5 \exp\left(-\frac{j_0-1}{j_0}\hat{Q}\hat{z}_{\rm SP}\right) = Q^5 \exp\left(-\left(Q/Q_s(x;A)\right)^{1-\frac{1}{\sqrt{\lambda}}-\sqrt{\frac{j_0-1}{\sqrt{\lambda}}}}\right) \tag{5.13}$$

$$= Q^5 \exp\left(-\frac{j_0-1}{j_0}\hat{Q}\left\{\frac{Q}{g_0^2 N_c \xi(j_0)\hat{S}\left(b\right)}\right\}^{-\frac{1}{\sqrt{\lambda}}} \times \hat{s}^{-\sqrt{\frac{j_0-1}{\sqrt{\lambda}}}}\right.$$

$$\left.\times\left(\frac{j_0-\frac{\sqrt{\lambda}\ln(1/\hat{z}_{\rm SP})}{\ln\hat{s}+\ln\hat{z}_{\rm SP}}}{\sqrt{\frac{2}{\pi\sqrt{\lambda}(\ln\hat{s}+\ln\hat{z}_{\rm SP})}}}\right)^{-\frac{1}{\sqrt{\lambda}}}\right)$$

In eq. (5.13) we chose $z^-_{\rm SP}$, since it gives a larger contribution. The saturation momentum is equal to

$$Q_s\left(x;A\right) = \left\{\frac{j_0}{j_0-1}\left(\frac{1}{x}\right)^{\sqrt{\frac{j_0-1}{\sqrt{\lambda}}}}\left(g_0^2 N_c \xi(j_0)\hat{S}\left(b\right)\right)^{\frac{1}{\sqrt{\lambda}}}\left(\frac{j_0-\frac{\sqrt{\lambda}\ln(1/\hat{z}_{\rm SP})}{\ln\hat{s}+\ln\hat{z}_{\rm SP}}}{\sqrt{\frac{2}{\pi\sqrt{\lambda}(\ln\hat{s}+\ln\hat{z}_{\rm SP})}}}\right)^{-\frac{1}{\sqrt{\lambda}}}\right\}^{\frac{1}{j(\lambda)}} \tag{5.14}$$

where

$$\text{with } j(\lambda) = 1-\frac{1}{\sqrt{\lambda}}-\sqrt{\frac{j_0-1}{\sqrt{\lambda}}}$$

From eq. (5.14) one can see that F_2 has a geometrical scaling behavior, if we neglect the log dependence of the saturation scale. The most interesting result is the fact that $Q_S \propto \left(S\left(b\right)\right)^{\frac{1}{\sqrt{\lambda}\,j(\lambda)}}\ (1/x)^{\sqrt{\frac{j_0-1}{\sqrt{\lambda}}}\,\frac{1}{j(\lambda)}}$. At very large λ, the saturation momentum is constant and does not depend on A and x. However, the A dependance is more suppressed, namely $A^{1/(3\sqrt{\lambda})}$, while the x dependence has a suppression, which is however a much weaker one $(1/x)^{\lambda^{-1/4}}$. Such a behavior is similar to what we expect in high density QCD for the running QCD coupling [32].

5.2 $z^2\, g_0^2\, s \ \geq\ 1$ but $b_0^2 \propto z^2\, s \leq 1/m_{\rm graviton} \leq R_A^2$

In this kinematic region we have to take into account the multi-graviton interaction in the nucleon scattering amplitude. At high energy, the exchange of one Pomeron (reggeized graviton) induces an imaginary part of the amplitude, as has been discussed, which increases with energy. Such an increase leads to a nucleon cross section of the order of $2\pi b_0^2(x)$, where b_0 can be estimated using the following equation

$$A_N^G(s,z,b_0) \ \approx\ 1/2 \tag{5.15}$$

The nucleon amplitude for single reggeized graviton exchange can be evaluated using eq. (5.2) and the fact that $u(b) \to b^2/(2zz')$ at large b. Repeating the same procedure, we obtain that (with $\hat{b} = b/z'$)

$$A_N^G(s,z,b) \ \xrightarrow{b^2\geq z^2<z'^2}\ \frac{\hat{z}}{\hat{b}^2}(\hat{s})^{j_0-1}\,\hat{z}^{j_0-1}\ \exp\left(-\frac{\sqrt{\lambda}\,\ln^2\left(\frac{2\hat{z}}{\hat{b}^2}\right)}{2(\ln\hat{s}+\ln\hat{z})}\right) \tag{5.16}$$

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From eq. (5.15) and eq. (5.16), we see that $\hat{b}_0^2 \approx \hat{s}\,\hat{z}^2$. Therefore,

$$F_2 \;\propto\; \exp\left(-\hat{Q}\hat{z} - Const\, S\,(b)\;\hat{s}^{j_0-1}\,\hat{z}^{j_0}\right) \tag{5.17}$$

which leads to small values of the typical $\hat{z} = \hat{z}_{\rm SP}$, namely,

$$\hat{z}_{\rm SP} \;=\; \frac{\left(Q/\hat{S}\,(b)\right)^{\frac{1}{j_0-1}}}{j_0\,\hat{s}} \tag{5.18}$$

At high energies, $z_{\rm SP}$ is small, and the nucleon amplitude turns out to be small even at small values of b. Therefore, we do not need to discuss this region separately, and the answer is just the same as in the previous section.

5.3 $z^2\,g_0^2\,s \;\geq\; 1$ but $b_0^2 \propto z^2\,s \geq 1/m_{\rm graviton} \leq R_A^2$

At such large impact parameters, we cannot use eq. (5.1) and eq. (5.5). The main contribution in this region stems from the exchange of the lightest hadron (in our case of the graviton) [31], which has the form given in eq. (2.15), and can be written as

$$A(s, b \gg z') \;\propto\; i\,g_0^2\,s\,z^4\,\exp\left(-m_{\rm graviton}\,b\right) \tag{5.19}$$

The typical impact parameter can be found from the equation $A\,(eq.\ (5.19); s, b) \approx 1/2$, which gives $b_0 \;=\; (1/m_{\rm graviton})\,\ln\left(g_0^2\,z^4\,s\,\right)$. Therefore for F_2 we have

$$\begin{aligned} F_2\left(Q^2, x = Q^2/s\right) \;&=\; C\alpha' Q^5 \int d^2b \int z^2 dz e^{-Qz} Re \\ &\times \left\{1 - \exp\left(i g_0^2 N_c \xi(j_0) 2\pi S\,(b)\,(1/m^2_{\rm graviton})\ln^2\left(g_0^2 z^4 s\right)\right)\right\} \end{aligned} \tag{5.20}$$

In eq. (5.20), the main contribution stems from $z \propto 1/Q$, which leads to

$$\begin{aligned} &F_2\left(Q^2, x = Q^2/s\right) \;= \\ &\quad = C\alpha' Q^2 \int d^2b Re\left\{1 - \exp\left(i g_0^2 N_c \xi(j_0) 2\pi S\,(b)\,(1/m^2_{\rm graviton})\ln^2\left(g_0^2 \frac{1}{Q^2 x}\right)\right)\right\} \end{aligned} \tag{5.21}$$

One can see, that $F_2 \;\propto\; \alpha'\,Q^2\,R_A^2\left(\ln\ln\left(g_0^2\;\frac{1}{Q^2\,x}\right)\right)^2$. However, such a behavior is valid only in the restricted kinematic region when $(1/m_{\rm graviton})\,\ln\left(g_0^2\;\frac{1}{Q^2\,x}\right) \;<\; R_A$. For higher energies, we loose all the specifications related to the nucleus, and the nucleus interacts as a proton would do, but with the coupling constant $g_0^2\,N_c\,A$.

6. DIS with nuclei: dipole model.

In QCD, the DIS cross section can be written as a product of two factors [28, 29, 15], namely the probability to find a dipole in the virtual photon, and the scattering amplitude of the dipole with the target. In this way the DIS cross section is given by the expression

$$\sigma_{\rm tot}\left(DIS; Q^2, x\right) \;=\; \int \frac{d^2r\,d\zeta}{2\pi}\,d^2b\,|\Psi\,(Q; r, \zeta)\,|^2\,N\,(r, b, x) \tag{6.1}$$

where N is the imaginary part of the scattering amplitude of the dipole with size r off the target, and ζ is the fraction of the energy carried by the quark of the dipole. We can try to generalize this equation to N=4 SYM, namely,

$$\sigma^A_{\rm tot}\left(DIS; Q^2, x\right) \;=\; \int \frac{d^2r\, d\zeta\, dz}{2\pi}\, d^2b\, |\Psi\left(Q; r, z, \zeta\right)|^2\, N_A\left(z, b, x\right) \tag{6.2}$$

The factorization of eq. (6.1) is valid on very general grounds (see ref. [21]), and should hold in any reasonable theory, since it is based on the structure of the interaction in time. In eq. (6.2), we use the fact that the interaction due to the graviton exchange does not depend on the size of the interacting particles, (see eq. (2.15)). We do not see any specific features for the dipole - target interaction, and thus we should be able to use for the nucleus amplitude (N_A) the formulae that we have discussed in the previous sections. On the boundary, $\Psi\left(Q; r, \zeta\right)$ is known and it is proportional to $K_1\left(\bar{Q}\, r\right)$, or to $K_0\left(\bar{Q}\, r\right)$, for different polarizations of the virtual photon with $\bar{Q}^2 = Q^2\zeta(1-\zeta)$. We can reconstruct $\Psi\left(Q; r, \zeta\right)$ using the Witten formula [24], namely,

$$\begin{aligned}&\Psi\left(Q; r, \zeta\right) = \\ &\frac{\Gamma\left(\Delta\right)}{\pi\Gamma\left(\Delta-1\right)}\int d^2r'\left(\frac{z}{z^2+(\vec{r}-\vec{r}')^2}\right)^{\Delta} K_0\left(\bar{Q}r'\right) \;\;\text{with}\;\; \Delta_\pm = \frac{1}{2}\left(d \pm \sqrt{d^2+4m^2}\right)\end{aligned} \tag{6.3}$$

Using the formulae **3.198**, **6.532(4)**, **6.565(4)** and **6.566(2)** from the Gradstein and Ryzhik tables, ref. [33], and using the Feynman parameter (t), we can rewrite eq. (6.3) in the form

$$\begin{aligned}\Psi\left(Q; r, \zeta\right) &= \frac{\Gamma\left(\Delta\right)}{\pi\Gamma\left(\Delta-1\right)}\int \xi d\xi d^2r' \frac{J_0\left(\bar{Q}\xi\right)}{\xi^2+r'^2}\left(\frac{z}{z^2+(\vec{r}-\vec{r}')^2}\right)^{\Delta} \\ &= \frac{\Gamma\left(\Delta+1\right)}{\pi\Gamma\left(\Delta-1\right)}\int \xi d\xi d^2r' \int_0^1 \frac{dt}{z} t^{\Delta-1}(1-t) J_0\left(\bar{Q}\xi\right) \\ &\quad\times\left(\frac{z}{tz^2+t(\vec{r}-\vec{r}')^2+(1-t)r'^2+(1-t)\xi^2}\right)^{\Delta+1} \\ &= \frac{\Gamma\left(\Delta\;+1\right)}{\pi\Delta\Gamma\left(\Delta-1\right)}\int \xi' d\xi' \int_0^1 dt t^{\Delta-1} J_0\left(\bar{Q}k\right)\left(\frac{z}{tz^2+r^2t(1-t)+\xi'^2}\right)^{\Delta} \\ &= \frac{1}{\pi 2^{\Delta-1}\Gamma\left(\Delta-1\right)} z^{\Delta}\int_0^1 dt\left(\frac{\bar{Q}^2}{z^2+(1-t)r^2}\right)^{\Delta-1} K_{\Delta-1}\left(\bar{Q}\sqrt{t(z^2+(1-t)r^2)}\right)\end{aligned} \tag{6.4}$$

Using eq. (6.4), we can rewrite eq. (6.1) in the form

$$\begin{aligned}\sigma_{\rm tot}\left(DIS; Q^2, x\right) = \int \frac{d^2r d\zeta}{2\pi} d^2b dz \Bigg\{ &\frac{1}{\pi 2^{\Delta-1}\Gamma\left(\Delta-1\right)} z^{\Delta}\int_0^1 dt\left(\frac{\bar{Q}^2}{z^2+(1-t)r^2}\right)^{\Delta-1} \\ &\times K_{\Delta-1}\left(\bar{Q}\sqrt{t(z^2+(1-t)r^2)}\right)\Bigg\}^2 Re\left(1-\exp\left(i\frac{g_0^2 N_c}{4}\frac{Q^2}{x} z^4 S\left(b\right)\right)\right)\end{aligned} \tag{6.5}$$

where we used the simple exchange of the graviton as in eq. (3.4). Using the asymptotical expression for the modified Bessel function, we can do the integral over z in saddle point

approximation, and the equation for the saddle point $z_{\rm SP}$ has the following form

$$\sum_{i=1}^{2} \frac{\bar{Q}\, t_i}{\sqrt{t_i(z_{\rm SP}^2\, +\, r^2)}}\, z_{\rm SP}\ +\ i g_0^2 N_c\, \frac{Q^2}{x}\, z_{\rm SP}^3\, S\,(b)\ =\ 0 \tag{6.6}$$

which leads to

$$z_{\rm SP}\ =\ \sqrt{\frac{\sum_{i=1}^{2} \frac{i\,\bar{Q}\, t_i}{\sqrt{t_i\, r^2}}}{g_0^2 N_c \frac{Q^2}{x}\, S\,(b)}}\ \propto\ \sqrt{\frac{x}{Q\, S\,(b)\, r}}\ \ll\ r \tag{6.7}$$

In eq. (6.5),eq. (6.6) and eq. (6.7), we introduced two variables, t_1 and t_2, to describe $|\Psi\,(Q; r, \zeta)\,|^2$. From eq. (6.7) and eq. (6.5), we obtain

$$\sigma_{\rm tot}\left(DIS; Q^2, x\right) = \int \frac{d^2 r d\zeta}{2\pi} d^2 b d\,(z - z_{\rm SP}) \int_0^1 \prod_{i=1}^{2} \frac{dt_i}{\sqrt{t_i}} \frac{\pi \Gamma^2\,(\Delta)}{\pi^2 \Gamma^2\,(\Delta - 1)} \left(\frac{z_{\rm SP} \bar{Q}^2}{r^2}\right)^{2\Delta - 2} \frac{r^3}{\bar{Q}} \tag{6.8}$$

$$\times \exp\left(-\sum_{i=1}^{2} \bar{Q} \sqrt{t_i r^2}\right) \left\{1 - \exp\left(i \frac{g_0^2 N_c}{4} \frac{Q^2}{x} S\,(b) \left(\frac{i \sum_{i=1}^{2} \frac{\bar{Q} t_i}{\sqrt{t_i r^2}}}{g_0^2 N_c \frac{Q^2}{x} S\,(b)}\right)^2\right)\right\}$$

Introducing a new variable $\tilde{Q}\ =\ \bar{Q}\ \sum_{i=1}^{2} \sqrt{t_i}$, we can integrate over r using the steepest decent method. The equation for the saddle point reads as follows

$$r_{\rm SP}\ =\ \left(\frac{i\, x}{2\, g_0^2 N_c\, S\,(b)\ \tilde{Q}}\, \frac{\tilde{Q}^2}{Q^2}\right)^{\frac{1}{3}} \tag{6.9}$$

and

$$\text{the second term in eq. (6.8)} = -\int \frac{r_{\rm SP} d\,(r - r_{\rm SP})\, d\zeta}{2\pi} d^2 b d\,(z - z_{\rm SP}) \int_0^1 \prod_{i=1}^{2} \frac{dt_i}{\sqrt{t_i}} \frac{\pi \Gamma^2\,(\Delta)}{\pi^2 \Gamma^2\,(\Delta - 1)}$$

$$\times \left(\frac{z_{\rm SP} \bar{Q}^2}{r_{\rm SP}^2}\right)^{2\Delta - 2} \frac{r_{\rm SP}^3}{\bar{Q}} \exp\left(-\left(-i \frac{Q^2}{Q_s^2}\right)^{\frac{1}{3}} + i \frac{3}{2} g_0^2 N_c \frac{Q^2}{x} S\,(b)\, \frac{1}{r_{\rm SP}^4} (r - r_{\rm SP})^2\right) \tag{6.10}$$

with

$$Q_s^2\ =\ \left(\frac{2\, g_0^2 N_c\, S\,(b)}{x\, \zeta^4 (1 - \zeta)^4 \left(\sum_{i=1}^{2} \sqrt{t_i}\right)^4}\right)\ \propto\ \frac{A^{\frac{1}{3}}}{N_c}\, \frac{1}{x} \tag{6.11}$$

Eq. (6.10) shows geometrical scaling behavior, at least to within exponential accuracy. The saturation momentum of eq. (6.11), has expected from the high density QCD A dependence, increases in the region of low x in the same way as for the DIS case, with the $\mathcal{R}$ current given by eq. (3.7). In general, eq. (6.10) displays the same features as eq. (3.5), (see also eq. (3.9)).

7. DIS in a shock wave approximation.

The approach developed above, has to be compared with ref. [27], in which DIS with a nucleus target was considered in the framework of the shock wave approximation. In

this paper the usual decomposition of the DIS cross section into two factors given by eq. (6.1)[28, 29, 15], which are the probability to find a dipole in the virtual photon, and the amplitude of the scattering of the dipole with the target, is used (see eq. (6.1)) where ζ is the fraction of energy carried by the quark of the dipole. In ref. [27] it is suggested to study the dipole-target amplitude in the semiclassical limit of the dipole scattering, in the presence of the shock wave that was produced by the nucleus, in the spirit of ref. [34]. In this approach, the dipole is located at the boundary of the AdS_5 space, and the two-dimensional surface of the string is characterized by $X^\mu = X^\mu(\tau,\sigma)$, (with $\mu = 0,\ldots,4$), which depends on two coordinates (τ,σ). The string Nambu-Goto action takes the following form

$$\begin{aligned} S_{\rm NG} &= \int d\sigma d\tau \mathcal{L} = \frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-det G_{\alpha,\beta}} \\ \text{where} \quad G_{\alpha,\beta} &= g_{\mu,\nu}\left(X\right)\partial_\alpha X^\mu \partial_\beta X^\nu, \quad \alpha,\beta = \sigma,\tau \end{aligned} \tag{7.1}$$

In the presence of the heavy nucleus, the free metric of eq. (2.18) has to be altered in order to take into account the energy-momentum tensor that describes the interaction of the dipole string with the nucleus. The modified metric is given by

$$ds^2 \;=\; \frac{L^2}{z^2}\left(-2dx_+dx_- \,+\, (dx_\perp)^2 \,+\, dz^2\right) \,+\, T_{--}\delta\left(x_-\right)dx_-dx_- \tag{7.2}$$

In eq. (7.2), we denote $x_\pm \;=\; \frac{x_0 \pm x_3}{\sqrt{2}}$, where x_0 is the time in the normal four dimensional space. $x_4 \equiv z$. In ref. [27], $T_{--}\;\mu z^2\delta(x_)$, suggested in ref. [35], is used. Using this assumption of ref. [27], the metric reduces to the expression

$$\begin{aligned} ds^2 &= \frac{L^2}{z^2}\left(2dx_+\,dx_- \,+\, \mu\, z^4\,\delta\left(x_-\right)\,dx_-^2 \,+\, (dx_\perp)^2 \,+\, dz^2\right) \\ ds^2 &= \frac{L^2}{z^2}\left(2dx_+\,dx_- \,+\, \frac{\mu}{a}\Theta\left(x_-\right)\,\Theta\left(a-x_-\right)\,z^4\,dx_-^2 \,+\, (dx_\perp)^2 \,+\, dz^2\right) \\ ds^2 &= \frac{L^2}{z^2}\left\{-\left(1-\frac{\mu}{2a}z^4\right)\,dt^2 \,+\, \left(1+\frac{\mu}{2a}z^4\right)\,(dx_3)^2 \,+\, (dx_\perp)^2 \,+\, dz^2\right\} \end{aligned} \tag{7.3}$$

where a is chosen such that $\mu/2a \,=\, s^2$, and $a \,\sim\, 2R_A\Lambda/p_+ \,\propto\, A^{1/3}/p_+$ (see ref. [27] for details). In the last line of this equation, we omit the theta functions, since we are looking for the solution which does not depend on time (static solution [27]). The static approximation is not well justified (see ref. [39], which appeared after the first version of this paper we put on the net). However, the exchange of gravito, which interacts with the energy-momentum tensor (see eq. (2.15)) and which is responsible for the mediation of the gravitational force, is taken into account in this approximation. As we mentioned above, the main goal of this section is to confront the Glauber-Gribov approach for dipole -nucleus scattering, described in the previous section, with the static solution in the SW approximation. Although at first glance the solution of ref. [27] does not reproduce the result of the Glauber-Gribov approach (see below), we will argue, that by changing the form of the Lagrangian of the string interaction with the nucleus, we are able to reproduce the Glauber-Gribov formula, in the static solution. Therefore, although it is plausible that one can learn some physics from the static solution, we believe however, that we can learn no more than is already derived from the Glauber-Gribov approach.

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Using eq. (7.1), eq. (7.3) and the following parameterization of X^μ, namely $X^0 = t$, $X^1 = x$, $X^2 = 0$, $X^3 = 0$ and $X^4 = z(x)$ as in ref. [27], the action S is found to be equal to

$$S \;=\; \int_0^{a\sqrt{2}} dt \int_{-r/2}^{r/2} dx\; \mathcal{L}^{\rm static} \quad \text{with} \quad \mathcal{L}^{\rm static} \;=\; \frac{\sqrt{\lambda}}{2\pi}\,\frac{1}{z^2}\,\sqrt{(1+z'^2)\,(1-s^2\,z^4)} \tag{7.4}$$

From the Euler-Lagrange equation, which has the form (for the static solution)

$$\frac{\partial}{\partial x}\,\frac{\partial\,\mathcal{L}^{\rm static}}{\partial z'} \;-\; \frac{\partial\,\mathcal{L}^{\rm static}}{\partial z} \;=\; 0, \tag{7.5}$$

as in ref. [27], the following solution is found

$$S\,(\mu) \;=\; \frac{\sqrt{\lambda}\,a}{\pi\,c_0\sqrt{2}}\,\left\{\frac{c_0^2\,r^2}{z_{\rm max}^3} \;-\; \frac{2}{z_{\rm max}} \;+\; \frac{2}{z_h}\right\} \quad \text{with} \quad c_0 \;=\; \frac{\Gamma^2\,(1/4)}{(2\pi)^{3/2}} \;\text{and}\; z_h = \frac{1}{\sqrt{s}} \tag{7.6}$$

while $z_{\rm max}$ is the solution to the equation

$$c_0\,r \;\;=\;\; z_{\rm max}\,\sqrt{1 \;-\; s^2\,z_{\rm max}^4} \tag{7.7}$$

The amplitude N in eq. (6.1) is equal to [27]

$$N\,(r,x) \;\;=\;\; Re\,\{1-\exp\,(iS(\mu))\} \tag{7.8}$$

Using eq. (6.1), the cross section for DIS has the form

$$\sigma_{\rm tot}\left(DIS;Q^2,x\right) = \int \frac{d^2r d\zeta}{2\pi}|\Psi\,(Q;r,\zeta)\,|^2 N\,(r,x) \propto \int \frac{d^2r d\zeta}{2\pi} K_0^2\left(\bar{Q}r\right) N\,(r,x) \tag{7.9}$$

In eq. (7.9), we omitted the integration over impact parameter, since in this simplified string approach we consider that the nucleus has the infinite extension in the transverse plane. As we have discussed above, such a simplified approach to the impact parameter dependence could cause a lot of difficulties, since DIS cross sections depend on the impact parameter distribution both in the nucleus and in the nucleon amplitude (see section 5). In eq. (7.9) we simplified the wave function of the photon, which is known, by replacing it by K_0, since at large values of $\bar{Q}^2 = Q^2\zeta(1-\zeta)$, both components of the photon wave function for transverse and longitudinal polarized photons have the same behavior $\exp\left(-2\,\bar{Q}\,r\right)$.

We expect that in DIS the typical r will be small, and therefore we try to find the solution to eq. (7.7) for which $m \;\equiv\; c_0^4\,r^4\,s^2 \;\ll\; 1$. In this case, in ref. [27] three solutions have been found, which correspond to three different Riemann sheets of the cubic root, and which can be characterized by the index $n = 0, 1, 2$. They are

$$z_{\rm max} \;\xrightarrow{m\to 0}\; = \;\begin{cases} 1/\sqrt{s} & \text{for} \quad n=0; \\ i/\sqrt{s} & \text{for} \quad n=1; \\ c_0\,r & \text{for} \quad n=2; \end{cases} \tag{7.10}$$

The solution with $n = 2$ is the only one that matches the Maldacena result [36], for which $m \to 0$. For this solution, we can take the integral over r in the second term of eq. (7.8) in eq. (7.9) by the steepest decent method, with the saddle point

$$r_{\rm SP} \;=\; \sqrt{-i\frac{\sqrt{\lambda}\,a}{2\sqrt{2}\pi\bar{Q}}} \tag{7.11}$$

One can see that

$$m_{\rm SP} \;=\; c_0^4\, r_{\rm SP}^4\, s^2 \;=\; \frac{c_0^4\,\lambda^2\,a^2}{8\pi^2\bar{Q}^2} \propto \frac{\lambda^2\,A^{2/3}}{\bar{Q}^2} \;\ll\; 1 \;\text{ for }\; \bar{Q}^2 \;\gg\; \lambda^2\,A^{2/3} \tag{7.12}$$

and, therefore, in the kinematic region $\bar{Q}^2 \;\gg\; \lambda^2\,A^{2/3}$, the second term of eq. (7.8) leads to an approach of the unitarity bound for the DIS cross section of the following form

$$\sigma_{\rm tot}\left(DIS; Q^2, x\right) \;=\; \int \frac{d^2r\,d\zeta}{2\pi}\,|\Psi\left(Q; r, \zeta\right)|^2 \;-\; \sigma_{\rm II} \tag{7.13}$$

$$\sigma_{\rm II} \;=\; \exp\left\{-\left(\frac{Q_s}{Q}\right)^{\frac{1}{2}}\right\} \tag{7.14}$$

where the pre-exponential factor can be easily calculated. The saturation momentum Q_s has the form

$$Q_s \;=\; \frac{\sqrt{\lambda}a\,Q^2\zeta(1-\zeta)}{2\sqrt{2}\,\pi} \;\propto\; \sqrt{\lambda}A^{1/3}\,x \tag{7.15}$$

At large values of Q, eq. (7.15) leads to a term of the order of $xA^{1/3}\lambda/Q_s$, which corresponds to the twist expansion, with the anomalous dimension $\gamma = 1/2$. The A dependence is in accordance with this as well [37], but the x dependence looks strange. $Q_s \to 0$ at $x \to 0$, and therefore the theory predicts that for low x and $\bar{Q}^2 \;\gg\; \lambda^2\,A^{2/3}$, the DIS cross section is very small.

It turns out that in the kinematic region $\bar{Q}^2 \;\ll\; \lambda^2\,A^{2/3}$, the $n = 0$ solutions gives the largest contribution. Indeed, inserting this solution in eq. (7.6), one can find the saddle point in the integration over r, which is equal to

$$r_{\rm SP} \;=\; -i\frac{\pi\sqrt{2}\bar{Q}}{c_0\,\sqrt{\lambda}\,a\,s^{3/2}} \;\propto\; i\frac{\bar{Q}}{\sqrt{\lambda}\,A^{1/3}\,\sqrt{s}} \tag{7.16}$$

Evaluating $m = c_0^4\,r_{\rm SP}^2\,s^2$, namely

$$m \;=\; \frac{4\pi^4\bar{Q}^4}{\lambda^2\,A^{4/3}} \;\ll\; 1 \tag{7.17}$$

one can see that in the region where $\bar{Q}^2 \;\ll\; \lambda^2\,A^{2/3}$, we are dealing with small values of m, and we can use the solution of eq. (7.10). Then $\sigma_{\rm II}$ in this case is proportional to

$$\sigma_{\rm II} \;\propto\; \exp\left(-i\frac{Q}{Q_s}\right) \;\text{ with }\; Q_s \;=\; 4\frac{c_0\,\sqrt{\lambda}\,a\,s^{3/2}}{\pi\sqrt{2}\zeta(1-\zeta)} \;\propto\; \frac{A^{1/3}}{x} \tag{7.18}$$

The saturation momentum in eq. (7.18) displays all the typical properties that we expect from high density QCD.

It is worthwhile mentioning, that the solution with $n = 1$ leads to $\sigma_{\rm II} \;\propto\; \exp\left(\frac{Q}{Q_s}\right)$, with the same saturation momentum, and it can be selected out since σ should be positive.

Both eq. (7.14) and eq. (7.18) have in common the fact that $z^4_{\rm max} s^2$ turns out to be much smaller than unity ($z^4_{\rm max} s^2 \;\ll\; 1$). It means that in the general equation for the action of eq. (7.4), we can consider $z^4 s^2$ to be small, and we expand the action with respect to this parameter. In this case the contribution at high energy can be reduced to the following action

$$S^{\rm eikonal} \;=\; \frac{\sqrt{\lambda}\, a\, s^2}{\sqrt{2}\pi} \int_{-r/2}^{r/2} dx\; z^2\, \sqrt{1\,-\,z'^2} \;=\; Const\; s\, A^{1/3} \int_{-r/2}^{r/2} dx\; z^2\, \sqrt{1\,+\,z'^2} \quad (7.19)$$

This action is closely related to the eikonal formula, as one can see from the second term of eq. (7.19). Solving the Euler-Lagrange equation of eq. (7.5), we find that

$$1 + z'^2 = \frac{z^4_{\rm max}}{z^4} \quad (7.20)$$

which leads to

$$z_{\rm max} = i \frac{\Gamma(1/4)}{\sqrt{\pi}\,\Gamma(3/4)}\,(r/2) \quad (7.21)$$

Evaluating the integration over x in eq. (7.19), we obtain the scattering amplitude in the form

$$\begin{aligned} N\,(r,s) \;&=\; Re\left\{1 \;-\; \exp\left(-Const\, A^{1/3}\, \left(\frac{\Gamma(1/4)}{\sqrt{\pi}\Gamma(3/4)}\right)^2\, s\, \left(\frac{r}{2}\right)^3\right)\right\} \\ &=\; Re\left\{1 \;-\; \exp\left(-\kappa\, A^{1/3}\, s\, r^3\right)\right\} \end{aligned} \quad (7.22)$$

where we have absorbed all constant factors in the factor κ. It is easy to see that eq. (7.22) leads to

$$\sigma_{\rm II} \;\propto\; \exp\left(-i\left(\frac{Q}{Q_s}\right)^{\frac{1}{2}}\right) \quad (7.23)$$

with Q_s given by eq. (7.18). The difference between eq. (7.23) and eq. (7.18), as well as the fact that eq. (7.14) does not hold, requires explanation. Referring back to eq. (7.19), one can see that implicitly in $S^{\rm eikonal}$, we neglected the part of the action of eq. (7.4) which does not depend on s. Since this contribution contains a factor of $a \propto 1/s$ in front, we can expect that this contribution is negligible at high energy. However, the integral over x can be divergent and compensates this smallness. In ref. [27], it was suggested that a subtraction in the action would cancel the divergence at $z \to 0$. The eikonal formula suggests a different type of remedy for this divergence, namely to introduce the action in the following way (compare with eq. (7.4))

$$S \;=\; \int_{-\infty}^{\infty} dt \int_{-r/2}^{r/2} dx\; \Delta\mathcal{L}^{\rm static} \;=\; \int_0^{a\sqrt{2}} dt \int_{-r/2}^{r/2} dx\; \Delta\mathcal{L}^{\rm static} \quad (7.24)$$

$$\text{with} \quad \Delta\mathcal{L}^{\rm static} \;=\; \mathcal{L}\,(T_{\mu\nu}) \;-\; \mathcal{L}\,(T_{\mu\nu} = 0)$$

which leads to

$$\Delta\mathcal{L}^{\rm static} \;=\; \frac{\sqrt{\lambda}}{2\pi}\,\frac{1}{z^2}\,\sqrt{(1+z'^2)}\,\Big\{\sqrt{(1-s^2\,z^4)}\;-\;1\Big\} \tag{7.25}$$

Eq. (7.24) has a simple meaning, which is that we need to subtract the term which is responsible for the movement of the string in empty space during the period of time of the interaction, from the interaction induced by the energy-momentum tensor of the nucleus. One can see that the solution with the action given by eq. (7.24) and eq. (7.25), reproduces eq. (7.23) (see appendix) .

Therefore, we can conclude that the shock wave approximation can be reproduced by the eikonal formula. It should be stressed that the eikonal formula is more general, since in the framework of this approach we are able to introduce the impact parameter dependence as well as the quantum corrections related to the reggeized graviton (Pomeron, see section 5). It is worthwhile mentioning that in eq. (7.18) the shock wave approximation leads to the same amplitude as eq. (6.10), in the dipole approach. However, it should be stressed that the main result of ref. [27], that the dipole amplitude at high energy has a form

$$N(r)\;\propto\;1-\exp^{-\,rQ_s}\;\text{with}\;Q_s\;\propto\;Const(x)A^{1/3} \tag{7.26}$$

holds in the approach with the action given by eq. (7.24). Indeed, this result does not depend on the modification of the Lagrangian since for $z^4S^3\;\gg\;1$ the action is the same in both approaches , namely,

$$S\;=\;i\int_0^{a\sqrt{2}}dt\int_{-r/2}^{r/2}dx\;\frac{\sqrt{\lambda}}{2\pi}\,s\,\sqrt{1+z'^2}\;\to\;i\,Const(x)\,A^{1/3}\,r \tag{7.27}$$

The last equation comes from the equation of motion which leads to $z'=0$. This saturation momentum $Q_s(A)\propto Const(x)A^{1/3}$ needs an explanation since it does not appear in the Glauber-Gribov approach. First, the contribution of eq. (7.26) to the DIS cross section (see eq. (6.1)) has the form

$$\begin{aligned}\sigma\left(DIS;Q^2,x\right)\;=\;\frac{\pi}{Q^2}\Big\{1+\frac{\pi^2}{16}\frac{Q_s(A)}{Q}{}_2F_1\left(\frac{3}{2},\frac{3}{2},2,\frac{Q_s^2(A)}{4Q^2}\right)\\ -{}_3F_2\left(\{1,1,1\},\left\{\frac{1}{2},\frac{3}{2}\right\},\frac{Q_s^2(A)}{4Q^2}\right)\Big\}\end{aligned} \tag{7.28}$$

For $Q\;>\;Q_s(A)\;\sigma\left(DIS;Q^2,x\right)\;\to\;\pi/Q^2$ which corresponds to 1 in eq. (7.26). If we replace K_0 by its asymptotic behavior $\sigma\;\propto\;(1/Q^2)\times(1/(Q+Q_s(A))$. The physical meaning of $Q_s(A)$ is rather obvious: during the passage of the dipole through the nucleus the transverse momentum (Q) can get an additional momentum ΔQ due to elastic rescattering with the nucleons , namely

$$\Delta Q\;\propto\;q_\perp^N\;\times\;\text{number of collisions}\;=\;\frac{1}{R_N}\,A^{1/3} \tag{7.29}$$

where $q_\perp^N\;=\;1/R_N$ is the typical transverse momentum for elastic scattering with one nucleon (R_N is the nucleon radius). In the Glauber-Gribov approach, however, $q_\perp^N\;\propto$

$1/R_A \to 0$ due to the nucleus form factor (see eq. (2.8)). In the shock wave approach the nucleus wave function has not been taken into account and nucleons can have unrestricted transverse momenta. Therefore, we consider this momentum as the artifact of the shock wave model in which, we believe, we need to specify the DIS as scattering with $Q > \Delta Q \approx Q_s(A)$ if we are interested in finding the total cross section. However, this $Q_s(A)$ can manifest itself in the inclusive production leading to the situation with two characteristic momenta that has been advocated in ref. [38]. It should be stressed that eq. (7.29) is written for a string with the fixed transverse coordinate (see eq. (7.4) $X^1 = x$, $X^2 = 0$). In the general case $\Delta Q^2 = \frac{1}{R_N} A^{1/3}$. The second comment on eq. (7.29) is that we considered the rescatterings which are instantaneous in accordance with the static solution. In the region $Q > Q_s(A)$, the contribution, given by eq. (7.28), is small and the value of the total cross section for DIS is determined by the saddle point approximation (see eq. (7.23)) which is the same both in the shock wave approximation and in the Glauber-Gribov approach.

8. Conclusions

It is our common wisdom nowadays that N=4 SYM , which can be solved at large coupling values, can provide us with some knowledge of what potentially lies in the confinement region of QCD. However, the first analysis of high energy DIS scattering, performed in refs. [3, 10], demonstrated that the high energy scattering in N=4 SYM looks quite different from what has been known so far. Contrary to the usual expectations based on perturbative QCD and the parton model, that the main process at high energies is multiparticle production, it was found in refs. [3, 10] that in N=4 SYM the major contribution originates from quasi-elastic scattering. This also contradicts what is known from data.

The goal of this paper was to develop the Glauber-Gribov description of DIS on a nuclear target within the N=4 SYM, which should help to see the key features of high energy scattering in a more transparent way. For this purpose we employed the eikonal approximation which has been developed for N=4 SYM in refs. [3 – 6, 10]. Our results can be summarized as follows.

1. We derived the Glauber-Gribov formula (see eq. (2.41) and eq. (3.5)), and showed that for the case of graviton exchange, this formula displays the same general properties, such as the geometrical scaling behavior, as in the case of the high density QCD approach.

2. We demonstrated that graviton exchange indeed leads to a total cross section which is dominated by quasi-elastic re-scatterings. However, we found that the quantum effects responsible for graviton reggeization give rise to an imaginary part of the nucleon amplitude. This imaginary part, enhanced by multiple interactions, results in a DIS which looks similar to one predicted by the high density QCD, (see figure 6).

3. We concluded that in N=4 SYM the impact parameter dependence of the amplitude is essential, and the entire kinematic region can be divided into three regions. In the first region ($z^2\, g_0^2 N_c \leq 1$), we can use the eikonal formula with a single graviton

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or reggeized graviton exchange for the nucleon amplitude. In the second kinematic region, $z^2\, g_0^2 N_c \geq 1$ but $b_0^2 \propto z^2 s < 1/m^2_{\rm graviton} < R_A^2$, the multi-graviton exchange in the nucleon amplitude may become important. However, we found that this is not the case and still the single graviton exchange dominates. In the third kinematic region ($z^2\, g_0^2 N_c \geq 1$ and $1/m^2_{\rm graviton} < b_0^2 \propto z^2 s < R_A^2$), the multi-graviton exchanges in the nucleon amplitude must be included, and the related modification to the amplitude are discussed in section 5.3.

In this paper, we considered mostly the DIS of the $\mathcal{R}$ current with the target. However, in the last two sections, we discussed the traditional approach to DIS based on the factorization given by eq. (6.1). We considered DIS in two different ways. In the first one we generalized the usual dipole formula to N=4 SYM. We derived the probability to find a dipole in the virtual photon, in AdS_5 space, and considered for the dipole scattering amplitude the eikonal formula. In the second approach, we revisited the shock wave approximation that has been developed for DIS in ref. [27], and we showed that in this formalism we can also use the Glauber-Gribov approach for DIS in the region of $r \approx Q/A^{1/3}\sqrt{s}$. However, the Glauber-Gribov approach suggests a different way to renormalize the interaction Lagrangian proposed in ref. [27]. After such modification of the original formalism of ref. [27], both approaches, namely, the dipole model and the shock wave approximation give the same result for $r \approx Q/(A^{1/3}\sqrt{s})$. We gave the interpretation of the appearance of the new saturation momentum $Q_s(A)$ that does not depend on energy [27] and argue that in the shock wave approximation we should consider only DIS with $Q > Q_s(A)$. For such large values of Q the shock wave approximation with our modified Lagrangian reproduces the same result as the Glauber-Gribov approach.

In general, we conclude that N=4 SYM does not lead to any obvious contradiction, either with the high density QCD, or with experimental data. Therefore, we hope to learn something valuable about the confinement region from the exact solution in N=4 SYM, relying on the AdS/CFT correspondence.

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A. Shock wave approximation for DIS with our hypohesis on renormalized Lagrangian

In this appendix we consider the shock wave approximation to DIS suggested in ref. [27], with our hypothesis on the renormalised Lagrangian. As has been mentioned, we assume that the static AdS_5 renormalised lagrangian is the regular AdS_5 lagrangian with a nucleus present, minus the vacuum AdS_5 lagrangian, where the nucleus is not present. The expression to such a renormalised lagrangian is given by the following expression

$$\mathcal{L}^{\rm ren} \;= \mathcal{L}\left(T_{\mu\nu}\right) \,-\, \mathcal{L}\left(T_{\mu\nu}=0\right) \;=\; \mathcal{L}^{\rm nuc} \,-\mathcal{L}^{\rm vac} \tag{A.1}$$

$$\text{where} \quad \mathcal{L}^{\rm nuc} \,=\frac{\sqrt{2\lambda}}{2\pi}\frac{1}{z^2}\sqrt{\left(1+z'^2\right)\left(1-s^2z^4\right)} \quad \text{and} \quad \mathcal{L}^{\rm vac}=\frac{\sqrt{2\lambda}}{2\pi}\frac{1}{z^2}\sqrt{\left(1+z'^2\right)} \tag{A.2}$$

The Euler - Lagrange equation for $\mathcal{L}^{\rm ren}$ takes the form;

$$\begin{aligned} & \frac{\partial\mathcal{L}^{\rm ren}}{\partial\, z} \,-\, \frac{\partial}{\partial\, x}\left(\frac{\partial\mathcal{L}^{\rm ren}}{\partial\, z'}\right) \;=0 \\ \Rightarrow \quad & \frac{\partial\mathcal{L}^{\rm nuc}}{\partial\, z} \,-\, \frac{\partial\mathcal{L}^{\rm vac}}{\partial\, z} \,-\, \frac{\partial}{\partial\, x}\left(\frac{\partial\mathcal{L}^{\rm nuc}}{\partial\, z'}\right) + \frac{\partial}{\partial\, x}\left(\frac{\partial\mathcal{L}^{\rm vac}}{\partial\, z'}\right) \;=0 \end{aligned} \tag{A.3}$$

The various terms appearing in eq. (A.3) can be calculated from eq. (A.1), namely

$$\frac{\partial\mathcal{L}^{\rm nuc}}{\partial z} \;= -\frac{2}{z}\frac{\mathcal{L}^{\rm nuc}}{\left(1-s^2z^4\right)} \qquad \frac{\partial\mathcal{L}^{\rm vac}}{\partial z} \;= -\frac{2}{z}\mathcal{L}^{\rm vac} \tag{A.4}$$

$$\begin{aligned} \text{and} \;\; \frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}^{\rm nuc}}{\partial z'}\right) \;&= \frac{\partial}{\partial\, x}\left(\frac{z'}{1+z'^2}\,\mathcal{L}^{\rm nuc}\right) \\ &= \left(\frac{z''}{1+z'^2} \,-\frac{2z'^2z''}{\left(1+z'^2\right)^2}\right)\mathcal{L}^{\rm nuc} \,+\, \frac{z'^2}{1+z'^2}\frac{\partial\mathcal{L}^{\rm nuc}}{\partial z} \\ &\quad + \frac{z'\,z''}{1+z'^2}\frac{\partial\mathcal{L}^{\rm nuc}}{\partial z'} \\ &= \left(\frac{z''}{1+z'^2} \,-\frac{z'^2z''}{\left(1+z'^2\right)^2}\right)\mathcal{L}^{\rm nuc} \,+\, \frac{z'^2}{1+z'^2}\frac{\partial\mathcal{L}^{\rm nuc}}{\partial z} \end{aligned} \tag{A.5}$$

$$\text{similarly} \;\; \frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}^{\rm vac}}{\partial z'}\right) \;= \left(\frac{z''}{1+z'^2} \,-\frac{z'^2z''}{\left(1+z'^2\right)^2}\right)\mathcal{L}^{\rm vac} \,+\, \frac{z'^2}{1+z'^2}\frac{\partial\mathcal{L}^{\rm vac}}{\partial z} \tag{A.6}$$

Plugging eq. (A.4), eq. (A.5) and eq. (A.6) into eq. (A.3) gives the result

$$\begin{aligned} \frac{1}{1+z'^2}\left(\frac{\partial\mathcal{L}^{\rm nuc}}{\partial\, z} \,-\, \frac{\partial\mathcal{L}^{\rm vac}}{\partial\, z}\right) - \left(\frac{z''}{1+z'^2} \,-\frac{z'^2z''}{\left(1+z'^2\right)^2}\right)\left(\mathcal{L}^{\rm nuc} \,-\, \mathcal{L}^{\rm vac}\right) \;&=0 \\ \Rightarrow \quad -\frac{2}{z}\frac{\mathcal{L}^{\rm nuc}}{\left(1-s^2z^4\right)} + \frac{2}{z}\,\mathcal{L}^{\rm vac} \,-\, \frac{z''}{1+z'^2}\left(\mathcal{L}^{\rm nuc} \,-\, \mathcal{L}^{\rm vac}\right) \;&=0 \\ \Rightarrow \quad -\frac{2}{z}\frac{\left(1-\sqrt{1-s^2z^4}\right)}{\sqrt{1-s^2z^4}} \,+\, \frac{z''}{1+z'^2}\left(1-\sqrt{1-s^2z^4}\right) \;&=0 \\ \Rightarrow \quad 2\left(1+z'^2\right)-z\,z''\,\sqrt{1-s^2z^4} \;&=0 \end{aligned} \tag{A.7}$$

Recall that one can express z'' as $(1/2)\,\partial z'^2/\partial z$, hence eq. (A.7) simplifies to

$$2\left(1+z'^2\right) \;=\; \frac{1}{2}\,z\,\frac{\partial z'^2}{\partial z}\,\sqrt{1-s^2z^4} \quad\Rightarrow\quad \frac{dz'^2}{1+z'^2} \;=\; \frac{dz}{z}\,\frac{4}{\sqrt{1-s^2z^4}} \tag{A.8}$$

Integrating between $z'(z)$ and $z'(z_m) = 0$, where z_m is an extremum point of the string, one arrives at the result

$$z'^2 \;=\; \left(\frac{z}{z_m}\right)^4 \left(\frac{1+\sqrt{1-s^2z_m^4}}{1+\sqrt{1-s^2z^4}}\right)^2 - 1 \tag{A.9}$$

From eq. (A.9), one can find that

$$H(\xi,\xi_m) \equiv \int_0^{\xi} \frac{d\xi'}{\sqrt{\left(\frac{\xi'}{\xi_m}\right)^4 \left(\frac{1+\sqrt{1-\xi_m^4}}{1+\sqrt{1-\xi'^4}}\right)^2 - 1}} \;=\; \sqrt{s}\,(x \;-\; r/2) \tag{A.10}$$

where $\xi = \sqrt{s}z$. In eq. (A.10), the half of the string where $z' > 0$ is chosen, and we integrated over x from $-r/2$ to x. The maximal value of $\xi = \xi_m$, can be found from the following equation

$$H(\xi_m,\xi_m) \;=\; -\sqrt{s}\,r/2 \tag{A.11}$$

We have not yet found the expression for the function $H(\xi_m,\xi_m)$ through known functions, but the figure of figure 8 and figure 9 demonstrates the behavior of this function. The key difference with the solution proposed in ref. [27] is the fact that $H(\xi_m,\xi_m)$ of eq. (A.11) has only one solution in the region of small ξ_m, while $H(\xi_m,\xi_m)$ of ref. [27] has two solutions (see figure 8). We can simplify the integrand by its expression at low ξ, namely,

$$H^{\text{Low }\xi}(\xi,\xi_m) \;\to \int_0^{\xi} \frac{d\xi'}{\sqrt{\xi'^4/\xi_m^4 - 1}} = \int_0^{\xi/\xi_m} \frac{d\zeta\xi_m}{\sqrt{\zeta^4-1}} \tag{A.12}$$

Changing the integration variable to $\zeta^2 = \sin\theta$, then eq. (A.12) becomes;

$$H^{\text{Low }\xi}(\xi,\xi_m) \;\to = \frac{i}{2}\,\xi_m \int_0^{\arcsin\left(\xi^2/\xi_m^2\right)} \frac{d\theta}{\sqrt{\sin\theta}} \tag{A.13}$$

Finally changing the variable of integration once again to $\sqrt{\sin\theta} = t$, eq. (A.13) reduces to

$$\begin{aligned} H^{\text{Low }\xi}(\xi,\xi_m) \;\to\; &= i\,\xi_m \int_0^{\xi/\xi_m} \frac{dt}{\sqrt{1-t^2}} \;=\; \text{ellpt}\left\{\arcsin\left(\xi/\xi_m\right),0\right\} \\ &= \sqrt{s}\,(x - r/2) \end{aligned} \tag{A.14}$$

where $\text{ellpt}(\phi,k)$ is the elliptic function defined as

$$\text{ellpt}(\phi,k) \;=\; \int_0^{\sin\phi} \frac{dt}{\sqrt{1-k^2t^2}\sqrt{1-t^2}} \tag{A.15}$$

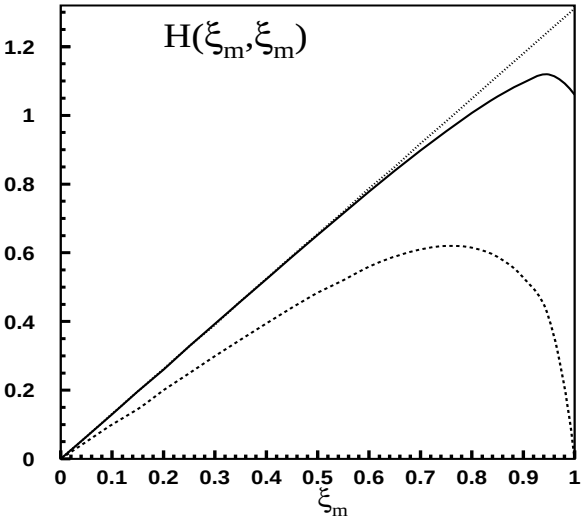

Figure 8: Function $H(\xi_m, \xi_m)$ versus ξ_m for small values of ξ_m. The solid line shows the function $H(\xi_m, \xi_m)$ given by eq. (A.10), and the dotted line is the same function for the solution given in ref. [27] while dashed line describes the approximation of eq. (A.12).

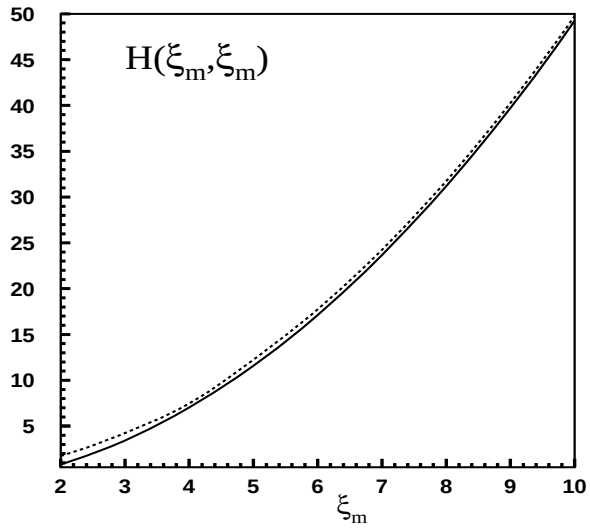

Figure 9: Function $H(\xi_m, \xi_m)$ versus ξ_m for large values of ξ_m. The solid line shows the function $H(\xi_m, \xi_m)$ given by eq. (A.10), dotted line is the approximation by eq. (A.16).

At large values of ξ, expanding the integrand at large values of ξ, we obtain

$$H^{\text{High }\xi}(\xi, \xi_m) \to i \frac{\xi_m^2}{\sqrt{2+2\sqrt{1-\xi_m^4}}} \int_0^{\xi} d\xi' = i \frac{\xi_m^2 \xi}{\sqrt{2+2\sqrt{1-\xi_m^4}}} = \sqrt{s}\,(x - r/2) \quad \text{(A.16)}$$

figure 8 and figure 9 show how the simplified equations (eq. (A.12) and eq. (A.16)), describe the exact function $H(\xi_m, \xi_m)$ given by eq. (A.10).

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Using eq. (A.12), one can easily see that eq. (7.4) with Lagrangian of eq. (A.1) reproduces eq. (7.23), which we obtain from the eikonal formula.

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